PAULO FREIRE AND MATHEMATICS, FOR A SITUATED APPROACH OF MATHEMATICS

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ABSTRACT

With a background on the discussions around the foundations of mathematics that took place in the late nineteenth, early twentieth century, in Europe and in the United States, we discuss the conception of pure, universal and neutral entities. We focus on the role of mathematics in giving support to this conception and we analyze the effects of such approach in Brazil, a huge country, out of developed centers, dominated by the contrasts of extreme situations. We consider the contributions of the Brazilian Educator Paulo Freire, whose practice as educator disregarded the directions imposed by the globalized world and has drawn attention to situated approaches, with the focus on the local things of life and people. We also argue that to the extent that Paulo Freire's pedagogy is committed to the dynamics of life, and not to the universal concepts, it crosses disciplinary boundaries, and therefore, embodies modes of thought of many disciplines, in particular in mathematics.

UNDERSTANDINGS OF MATHEMATICS: A FEW WORDS FROM THE DEVELOPING WORLD

We propose an understanding of mathematics as an interweaving with things of life. We escape from mathematics as a stabilized field of knowledge, with defined boundaries that make possible to take apart what is and what is not mathematics. We propose to approach mathematics with such a close adhesion to life that the design of its borders becomes blurred. It is no longer possible nor important to distinguish precisely mathematics among the usual categorizations of knowledge as disjoint fields, thus, weakening the conception of mathematics as an autonomous field of knowledge.

These ideas result from reflections on how to live and how to act in Brazil, a periphery country (outside the centers of power and the developed world). Faced with shortages of minimum conditions, Brazilian people invent (mathematical) solutions, as means of survival. They can not paralyze so they act, proposing surprising ways to deal with difficulties of any order to overcome the absence of given solutions. It has been this way since the beginning of the colonization: the clash of indigenous peoples, enslaved Africans and the European colonizer gave birth to a new people, which, as the Brazilian anthropologist Darcy Ribeiro observed, appeared “to the European eyes like bizarre people, what, added to our Indian tropicality, reaches the same eyes to make us exotic” (Ribeiro, 19…,p.66). This “bizarre singularity” formed a Brazilian way of existing that extends to all dimensions of living and thinking. Why would not extend to mathematics? Thus, our mathematical singularity cannot be separated from the things of life, and therefore does not show exactly the constructs of another mathematics whose historical path unfolded elsewhere. We cannot perceive this mathematics of life trying to find in it the purified constructs of the hegemonic math: its measurements, its numbers, its equations, its algorithms. This is the case of the mathematics that is practiced in indigenous, quilombos (communities of descendants of fugitive slaves) or illiterate communities, but also in cases where the hegemonic mathematics does not serve to the specific conditions of a local, time or situation (Cafezeiro, Kubrusly & Marques, 2017).

Just as life builds its paths and does not become paralyzed in the face of scarcity, the mathematics entangled in life is not surprised by the incompleteness. The mixture provides the encounter of the different. From this happens the collaboration, enhancing the invention of means to act in the face of uncertainty and scarcity.

For a situated approach to mathematics

> “Pay attention, the most important thing for Brazilians is to invent the Brazil that we want.”
>
> Darcy Ribeiro

We propose to keep a constant attention to the lived experience of people in some given place and date. A close adherence to life situations makes possible to understand the expression of these situations in a specific (abstract) jargon and the construction of concepts to propose solutions for these life problems together with the necessary mechanisms to shape them to the changings of life. It also makes possible the translation of concepts conceived in other domains or with other purposes to collaborate in solving these situated problems. This, we consider a “mathematical experience”: a constant path between concrete and abstract to conceive solutions.

Differing from what we usually consider mathematics, many times a situated mathematics is not recognized as mathematics for those who have in mind the hegemonic constructs. Many times, although recognized, a situated approach is seen as under-developed mathematics. This kind of analysis is a consequence of disregarding the capacity of attendance to local demands and prioritizing the equalization to the hegemonic centers, what results in taking the hegemonic mathematics as a reference of development.

In some few cases around the world where educational policies take into account the cultural identities to propose specific practices of mathematics to specific cultures, we can witness the accusations of educational failure. We must observe that the criteria of success (national exams, school tests) attend to the hegemonic conception of math. They are usually based on the reproduction and application of hegemonic mathematics concepts to solve problems that are considered universal. The criteria of success does not usually give visibility to the creative capacity of proposing solutions to local issues.

The attentive look to the situations in a given place, at a given time, put a light on the process of reshaping hegemonic concepts to better fit the solutions of local problems according to the local cultural background. This causes a blending of knowledge from which result new constructs, a renewal of concepts. This does not mean to make concessions to subjugated cultures. Instead, this is an open path to re-creation. We agree with Law (1977) that traduction is also trahison: the recognition that knowledge undergoes changes in order to become useful to the demands of life. This undermines the conception of pure entities, stabilized, neutral and universal, even in mathematics. It is precisely this recognition of traduction that shows that the situated approaches are not a rejection of the hegemonic knowledge. Actually, it enhances the possibilities of collaboration between hegemonic and local knowledge.

A situated approach diverges from learning based on assimilation and reproduction. Around the 1970s, Bourdieu (1975) realized that the French educational system was based on this practice of assimilation and reproduction. He argued in his "The Reproduction" that the French educational system reproduces internally the power relations by means of a symbolic violence. Before him, between 1929 and 1935, Gramsci had already noticed that the (Italian) educational system favored the children of privileged classes: they "*breathe in*, as the expression goes, a whole quantity of notions and attitudes which facilitate the educational process properly speaking" (Gramsci, 1998, p. 172). This theme occupied his mind during the period he was in fascist prison and led him to propose an educational model seeking to escape of "the general and traditionally unquestioned prestige of a particular form of civilization"(Gramsci, 1999, p. 166). The basic principle of his proposal was the perception of the historical character of knowledge, which implies recognizing human being in his time and in the place where he lives.

At about the same period as Bourdieu's "The Reproduction", the Brazilian educator Paulo Freire was exiled in Chile because of the military coup of 1964. The accusation against him was due to his practice with adult literacy, a subversive practice, under the eyes of the repressive government. There, he published "Pedagogy of the Oppressed", where he observed in the Brazilian educational system a similar situation to that perceived by Bourdieu in France. He referred to a "banking account educational system" (Freire, 1987, p. 33), a practice where the educator assumes the role of transferring his knowledge to the learner, also an educational proposal based in assimilation and reproduction. According to this practice, knowledge is in the first place, as something finished and out of question, just as a banking system, where the priority is the capital that circulates to be deposited in bank accounts. As Bourdieu, Paulo Freire noticed that this reserves to the learners the role of assimilating the transferred knowledge, thus, ignoring their creative potential. As Gramsci, Freire realized that this practice disregards the historical character of knowledge construction and learning.

Paulo Freire kept his attentive eyes to the Brazilian reality, and proposed educational practices based on the problems lived by a people in a time. A situated approach emerged, with the focus in the dynamic of transformation and construction, where knowledge is not separated from its construction path. This is a less authoritarian and more libertarian educational practice.

In this paper, we show how the educative proposals of the Brazilian educator Paulo Freire reinforces the practice of a situated approach for education. We show not only that the proposals of Paulo Freire contribute to the understanding of mathematics, but how these proposals are themselves mathematical.

UNDERSTANDINGS OF MATHEMATICS: PARADOXES, UNDECIDABILITY, ORACLES

At the turn of the nineteenth to the twentieth century, a group of mathematicians was concerned with the understanding of the foundations of mathematics. Great advances that had been made in the previous century motivated the efforts to ensure the accuracy and absolute confidence in mathematical results. Also, the modern conception of science settled from the seventeen century in Cartesian bases motivated the establishment of mathematics as an objective field, able to give support to all sciences. The mathematical scene was dominated by a conception of mathematics as a trustable and completed field, to ensure the truth and the correct reasoning "If mathematical thinking is defective, where are we to find truth and certitude?" (Hilbert,1925)

In 1900, in a talk at the 2nd International Congress of Mathematicians in Paris, David Hilbert invited the collective of mathematicians to a joint effort towards the search of a solution for 23 problems which had not yet been proven. No result expressed mathematically should remain unproved.

A few years later, in 1902, Bertrand Russell was surprised and sought explanations for his perception that there was a paradox in set theory. He expressed his astonishment in a letter to Gottlob Frege (Van Heijenoort, 1967, pp. 124-125): "Let w be the predicate: 'to be a predicate that *cannot* be predicated of itself'. Can w be predicated of itself? From each answer its opposite follows."

This construction employs a self-referenced sentence, with a negation. The paradox arises when this sentence is applied to some expression of itself. Russell suggested to Frege that the same thing could be expressed in terms of sets: "Likewise, there is no class (as a totality) of those classes which, taken as a totality, do not belong to itself". In this case, the self-referenced sentence with a negation can be expressed as the set formed by all sets that does not belong to themselves. Let us name this set R. The paradox arises when we inquire: R belongs to R? "From each answer its opposite follows".

When expressed as sets, this self-referenced sentence with a negation allows us to see that this construction suggests a loop: Given a set, say $\{1\}$, we may check if it belongs to itself. We see that it does not, otherwise the set $\{1\}$ would be $\{1,\{1\}\}$. Now, we may check if $\{1,\{1\}\}$ belongs to itself. This would generate $\{1,\{1\},\{1,\{1\}\}\}$. With this construction, we escape from the contradiction, but it unfolds in an infinite construction. In order to rid Set Theory of paradox, Russell (1908) outlined his "Doctrine of Types", separating mathematical entities in a hierarchy of types, where each type would be a construction over the previous type. He explained:

> A *term* or *individual* is any object which is not a range. This is the lowest type of object. (…) The next type consists of ranges or classes of individuals. (…) The next type after classes of individuals consists of classes of classes of individuals. (…) A new series of types begins with the couple with sense. (…) Thus we obtain an immense hierarchy of types, and it is difficult to be sure how many there may be. (Russell, 1908, p. 497)

The perception of a paradox within mathematics was a sign in the opposite direction to the safe, consistent and complete math. Despite of this, the mathematicians of that century proceeded in the search of strong foundations to mathematics. Russell, himself, was engaged in a project to formulate all mathematics in logical terms.

In 1928, at the Bologna International Conference of Mathematicians, Hilbert presented what was later called the "Hilbert program". He proposed the formalization of mathematics to ensure the accuracy of all mathematical construction so that for each sentence written in a formal language, it would be possible to find a proof of its truthfulness or falsity.

For Hilbert, a formal system should meet three requirements: be complete, consistent and decidable. Completeness means: be able to demonstrate all assertions (or their negations) expressed in the system language. Consistence means: free of contradiction, and decidablility means the existence of a mechanical process able to check if a given formal sentence is true or false. Hilbert program was settled as a strong option, over the other approaches of mathematic foundations.

However, in 1930/31 the mathematician Kurt Gödel published his incompleteness theorems, indicating the impossibility of a formal system (expressive enough to formalize all arithmetic) to be both complete and consistent (Gödel, 1965, pp.5-6). To do so, he was inspired by the same class of paradoxes that Russell and other mathematicians had been considering since the end of the nineteenth century. Gödel dropped the first two issues of Hilbert's program.

As a reaction to the incompleteness, mathematicians engaged in a search for formalization of the concept of "machine", or "mechanical". This would provide an understanding of the scope of mathematics. In 1936, the mathematician Alan Turing conceived an abstract machine, which we now call the Turing machine. From this, Turing realized that a mechanical process could enter into an infinite loop. In this case, it would be impossible for another mechanical procedure to tell whether that first one would be in loop. This problem, which is now called "the halting problem", shows the impossibility of the existence of a decision procedure in Hilbert's conception.

Therefore, he dropped the last point of Hilbert's program, the impossibility of existence of a mechanical way of deciding every mathematical issue (Turing, 1936,pp. 230-231).

Besides the immediate results that Turing reached with the machine, an issue often goes unnoticed. It is about the impossibility of an exact mapping between an object (an intuition) and its representation. In Law (1997) words "all representation also betrays its object", and this was clear for Turing in 1936:

> No attempt has yet been made to show that the "computable" numbers include all numbers which would naturally be regarded as computable. All arguments which can be given are bound to be, fundamentally, appeals to intuition, and for this reason rather unsatisfactory mathematically. The real question at issue is "What are the possible processes which can be carried out in computing a number?"(Turing, 1936,p.249)

Aware of the impossibility of accurately represent the intuition of what would be "computable numbers", he assumed a quite different way of constructing and exposing his ideas and results. Contrary to mathematical practice of the time, he left apparent the inspirations and reasons that led him to each mathematical decision. Everything is justified in terms of the human actions to calculate a number, involving the pencil, paper, and human needs and limitations. Many examples in the paper illustrates this way of constructing mathematics. We shall transcribe a few:

After examining the human process of calculating, Turing decided to construct a machine according to the observed actions of what he named 'computer', a human being at the action of computing: 'We may now construct a machine to do the work of this computer'. (Turing, 1936,p.231)

Turing starts an explicit correspondence between the material arrangement constituting the computer (human actions + paper + pencil) and the machine mechanisms that he proposes. This correspondence is accurate to the point of considering situations where the human component of this arrangement raises, interrupting calculations and then resume:

> It is always possible for the computer to break off from his work, to go away and forget all about it, and later to come back and go on with it. If he does this he must leave a note of instructions (written in some standard form) explaining how the work is to be continued. This note is the counterpart of the "state of mind". (Turing,1936,p.249)

Turing justifies the choice of having just one symbol in each square tape in a limitation of human vision:

> The differences from our point of view between the single and compound symbols is that the compound symbols, if they are too lengthy, cannot be observed at one glance. This is in accordance with experience. We cannot tell at a glance whether 9999999999999999 and 999999999999999 are the same. (Turing,1936,p.250)

Later, in 1950, when the material resources to calculate were beyond paper and pencil, and computing machines were already there, Turing added a desk machine to his presentation of digital computers. This shows how seriously he took the observation of things in his life and his time, and the effort to link his mathematical work with the time and place where he lived. He cited the technology of the new time and showed that it did not bring incompatibilities with respect to his 1936 approach:

> The idea behind digital computers may be explained by saying that these machines are intended to carry out any operations which could be done by a human computer. The human computer is supposed to be following fixed rules; he has no authority to deviate from them in any detail. We may suppose that these rules are supplied in a book, which is altered whenever he is put on to a new job. He has also an unlimited supply of paper on which he does his calculations. He may also do his multiplications and additions on a "desk machine," but this is not important. (Turing, 1950,p.436)

Turing's mathematics leaves visible its process of construction, its links with the time and the place where it was conceived. Unlike the mathematical practice of that moment, he made apparent the links between the abstract construction and the world where he lived. This is a situated approach.

In 1938, Turing defended his PhD. In a straight adherence to Gödel’s research, he proposed a mechanism to complete formal systems. According to him, the fundamental idea came from Gödel: from the awareness of incompleteness, the construction of an infinite search for completeness.

> The well known theorem of Gödel shows that every system of logic is in a certain sense incomplete, but at the same time it indicates means whereby from a system L of logic a more complete system L' may be obtained. By repeating the process we get a sequence L, L1 = L', L2 = L1' , L3 = L2',. . . of logics each more complete than the preceding. (Turing, 1938, p.1)

The mechanism consisted in introducing in the system, as an axiom, a statement that the system could not reach. Proceeding in this way, step-by-step, he formed a hierarchy where each system is a bit more complete than the previous. The *motum perpetuum* comes from the awareness of incompleteness. This produces an infinite search for completeness, even knowing that completeness would never be reached, as Gödel had shown.

It is not surprising that Turing's approach passed by an infinite search, a hierarchy, as in Russell's type theory, to escape from the paradox. Gödel had commented in his paper the analogy between his approach and paradoxes of the kind of Russell's paradox: "The analogy between this result and Richard's antinomy leaps to the eye; there is also a close relationship with the "liar" antinomy" (Gödel,1965,p.9).

When constructing his hierarchy, Turing had to deal with those problems that in 1936 he had shown to be not solvable by a machine, as the halting problem. He, thus, introduced in mathematics something unexpected. Something that in no way resembled the entities admitted so far as mathematics: an oracle (the o-machine): 'some unspecified means of solving number-theoretic problems, a kind of oracle as it were. We shall not go any further into the nature of this oracle apart from saying that it cannot be a machine". (Turing, 1938,p.18)

The oracle is an entity that, by some process not described mathematically, would provide the result of non-computable step. Turing again started a different mathematical practice able to host and (co) operate with entities whose descriptions are beyond the mathematical universe. It is not the purified mathematics, but the possibility to operate without distinction with computable and non-computable, mathematical and non-mathematical.

This historical path made clear two points that are the basis of our research:

The first is the observation that, by itself, mathematics is unable to solve problems within its own scope. Hence, the need for a mathematics that is not immobilized by contradictions, and can act even in a context of incompleteness. Incompleteness suggests that mathematics requires the collaboration of multiple and heterogeneous agents, invoking a knowledge that is quite different of what is usually said to be mathematical knowledge. This was the way indicated by Turing in 1938 with the o-machine.

The second is the observation that entities acquire a neutral and universal appearance when their links with the things in life are omitted. A-historical entities, that is, entities that do not show their construction process, appear to have an autonomous existence, independent of things in the world. In this way, they are out of questions. This point is strengthened by the way that Turing constructed his mathematics, always making clear the links with life. With this, his proposals achieved a general acceptance, even by those mathematicians that were proposing their own solutions for the same problem. For example, Gödel asserted: "The most satisfactory way, in my opinion, is that of reducing the concept of finite procedure to that of a machine with a finite number of parts, as has been done by the British mathematician Turing" (as cited in Soare, 2007,p.714). Kleene said: "Turing computability is intrinsically persuasive. λ–definability is not intrinsically persuasive. General recursiveness scarcely so" (as cited in Soare, 2007,p.714).

Bearing in mind that mathematics requires other kinds of knowledge and mixes with them to form its own concepts, we argue in favor of a *situated* mathematics. The term 'situated" adopted here comes from the field of Sociology of Knowledge and Science and Technology Studies, a proposal summarized by Shapin (2010) in his long tilte: "Never Pure: Historical Studies of Sciences as if it was Produced by People with Bodies, Situated in Time, Space, Culture, and Society, and Struggling for Credibility and Authority". We also refer to Donna Haraway and her situated approach to feminist studies:

> We seek not the knowledges ruled by phallogocentrism (nostalgia for the presence of the one true World) and disembodied vision. We seek those ruled by partial sight and limited voice - not partiality for its own sake but, rather, for the sake of the connections and unexpected openings situated knowledges make possible. Situated knowledges are about communities, not about isolated individuals. The only way to find a larger vision is to be somewhere in particular. (Haraway, 1988, p.580):

Concerning mathematics, we refer to David Bloor and his proposal of the Strong Program of Sociology of Knowledge, where he defends for a historical approach to mathematics that leaves apparent the construction process of mathematical results, thus moving away from unquestionable and stable mathematical entities (Bloor, 1991). Concerning education, we refer to Downey & Lee (2009) talk in favor of a situated approach for learning, proposing the critical participation in engineering studies. Concerning case studies about learning experiences and different approaches to numbers, we refer to Verran (2001), focusing the understanding of the number system of a Yoruba Nigerian collective. With these directions in mind, we propose a dialogue with Paulo Freire's pedagogical proposal and the current situation of teaching and possibilities of construction and understanding of mathematics in Brazil.

We now turn to some comments about Brazil and the mathematics that is often practiced there.

THE PLACE FROM WHERE WE SPEAK

Brazil is a country of continental dimensions (twice the size of India). It was colonized by the Portuguese from 1500. They established the Catholic religion, the Portuguese language, and the European way of operating, ignoring the diversity of peoples and cultures that have lived in this place. At different times, the colonization process spread forming the Brazilian territory. Brazil received a large number of people kidnapped from Africa by the Portuguese and made them slaves. It also received a large amount of European and Asian immigrants. Today, Brazil is a country of diverse cultures, diverse landscapes, diverse climates, since its territory goes from above the equator to below the tropics. It is also a place of contrasts. Extreme concentration of wealth lives beside extreme poverty.

Despite all this diversity, it is usual in Brazil the adoption of global measures and universal mechanisms, often designed under a foreign perspective. There is a search for complying with foreign standards, what is justified on the need to acquire visibility in the international arena and achieving developed countries. Education, for example, is regulated by parameters that apply to the whole territory, from the huge metropolis São Paulo to Betânia, a small village in the Amazon Forest. There is a national test, the ENEN (National Examination of Secondary Education) that evaluates the student's knowledge in order to regulate the access into the university. It also ranks the education quality of schools.

According to a special volume about education in a Brazilian magazine named “Revista Trip” (Monteiro, 2011) the school Pedro I, in the village of Betânia, in the higher part of Solimões river, state of Amazonas, was classified in 2009 by the ENEM as the worst school in Brazil. The article adds details that *situates* this result: People of Betânia speak Ticuna, a native language. They also speak Spanish, because of the contact with Colombians through the forest. Thus, Portuguese is the third language; the natives are not fluent in Portuguese. According to the article, teachers complain about the natives” resistance to attend classes in Portuguese:

> We have to take it easy because students get angry. Natives have a short fuse. (…) They just want classes in Ticuna. But there is no teaching materials in Ticuna. Crazy it here. We insist that the Portuguese have to be the spoken language in school. But they are offended. They think that we are putting down their language. (Monteiro, 2011)

Referring to the exam, the natives complained: “I can not understand. It had to be in Ticuna.” With about 38,000 of Ticuna speakers, the natives claim a Ticuna school and University. They have a rich cultural production, including books written in Portuguese present to Brazilians the Ticuna culture (OGPTB, 2000), and they are organized in an association named General Organization of Ticuna Bilingual Teachers.

This example goes beyond the problem of language. It leads us think about global parameters, general strategies, measures that do not leave apparent any trace of the place where they were conceived, thus suggesting that they are suitable for any situation. We question the effects of following a knowledge that is said neutral and universal, and how this knowledge may act in a place like Betânia, disregarding all the particularities and local potential.

Global measures and universal mechanisms concerning education are present in a large scale project that is underway in Brazil, promoted by the Ministry of Education. The project aims to set “common national basis” for the school curricula, what means a fixed content that must be taught in each year in every school in Brazil. This certainly favors the textbook market, but not the local education: “teaching materials must undergo significant changes”, is mentioned on the first page of the introductory texts (MEC, 2015).

The priority to the imposition of a common national strategy embodied in a list of contents shows an educational policy strongly committed to a universal standard of quality. The list of contents acts as a way of controlling what happens in each school trying to ensure that things go in the “right direction”, that is, follow the ‘success” of the developed world. What is certainly out of this strategy are the local interests. The respect for the culture and regional singularities requires flexibility, freedom and autonomy in each school, just the opposite of what is proposed by this project. In place of the rigidity of a fixed curriculum, the respect for the culture and regional singularities demand monitoring and training of local teams of teachers, attendance in schools, encouraging the production of local materials (as the Ticuna culture of books).

The control set by a curriculum in national common grounds is the result of a national educational policy where ‘social issues” and “technical issues” are treated as disjoint issues. The first is ruled by the National Plan of Education (MEC,2014) and the second is ruled by this mentioned project, the Curricular Common National Basis (MEC, 2015), which promises to save Brazilian education with technical arrangements. This division disguises the evidence that very low salaries and absence of minimum conditions of work and learning invalidate the most refined curriculum proposal.

Despite the emphasis on the interaction between areas of knowledge (the introductory text of mathematics highlights “the necessary rapprochement between the mathematical knowledge and the local cultures”), mathematics seems to be an isolated item from the others (which are: language, natural sciences and human sciences), supported in its own rationality. The referred “rapprochement” would be obtained by contextualizing the pure and universally accepted mathematics in examples that, supposedly, would bring the local reality (as if it were possible to have a single “local” in such a vast country as Brazil). But the commitment with a knowledge produced elsewhere induces to weird contextualizations, attempts to construct meaning for issues that did not arise from local demands, but are worldwide imposed to students of a certain age. This goes in an opposite direction of recognizing regional differences from which local knowledge can be constructed.

Mathematics gives support to these approaches when its concepts, as numbers, statistics, measures, are taken as exact, suitable for any situation, presented without history. As autonomous entities, without process of construction, this knowledge is untouchable, free from any discussion.

THE LANGUAGE OF COLLECTIVES

Some trends in the philosophy of mathematics support the idea of thinking in mathematical entities as having an autonomous existence. As argued Paul Bernays (1935), a mathematician that was a collaborator of David Hilbert:

> (…) the tendency of which we are speaking consists in viewing the objects as cut off from all links with the reflecting subject. Since this tendency asserted itself especially in the philosophy of Plato, allow me to call it "Platonism". (Bernays, 1935, p.2)

Bernays defended that "the value of platonistically inspired mathematical conceptions is that they furnish models of abstract imagination. These stand out by their simplicity and logical strength. They form representations which extrapolate from certain regions of experience and intuition." (Bernays, 1935, p.3) As we can see, in this conception, links with the things of life act as a repression to the flow of abstract thinking.

In Brazil, most of the mathematics taught in schools is in tune with this conception. Operations, formulas, algorithms, and results are presented without discussions about their construction process, since they are already stabilized in global mathematics. This motivates to think that mathematical entities have an autonomous existence. The weight of a mathematics built over a history of many centuries naturalizes concepts and inhibits the questions.

Contrary to this, we come with two explanations reinforcing the view that, like any other knowledge, mathematics is born stepped in worldly things. We start by a sociologist of knowledge, Ludwik Fleck, in the thirties, and then we turn to a philosopher of mathematics, Bertrand Russell, in the early twentieth century. Fleck said:

> There is no emotionless statement as such nor pure rationality as such. How could these states be established? There is only agreement or difference between feelings, and the uniform agreement in the emotions of a society is, in this context, called freedom from emotions. This permits a type of thinking that is formal and schematic, and that can be couched in words and sentences and hence communicated without major deformation. The power of establishing independent existences is conceded to it emotively. Such thinking is called rational. (Fleck, 1935, p. 49)

For Ludwik Fleck, abstract entities (such as those that appear in mathematical discourse as independent existences) result from a process of purification that comes from an agreement, which, once cleaned of feelings, generates the objective, impartial and universal rational discourse. Fleck understands that the rational, objective reasoning appears when you hide subjectivity and emotions. Then everything seems to be a schematic chain of reason. There is thus a social component in the basis of what is called rational thinking.

From Bertrand Russell, we bring an excerpt where he justifies the use of induction under the basis of a collective experience, and *not* as a rational chain of steps. Induction is a way or reasoning to make generalizations: to extend to a broad domain a property that has been repeated a certain amount of times. In the following quote note that Russell replaces the certainty of a proof by 'some reason in favour of".

> But the real question is: Do *any* number of cases of a law being fulfilled in the past afford evidence that it will be fulfilled in the future? If not, it becomes plain that we have no ground whatever for expecting the sun to rise tomorrow, or for expecting the bread we shall eat at our next meal not to poison us, or for any of the other scarcely conscious expectations that control our daily lives. It is to be observed that all such expectations are only *probable*; thus we have not to seek for a proof that they *must* be fulfilled, but only for some reason in favour of the view that they are *likely* to be fulfilled. (Russell,1912, chapter VI)

Essentially, both Fleck and Russell seemed to agree that what we usually take as a pure abstract (objective) thought is, in fact, an assemblage in which worldly things are not of minor importance.

When we renounce the universal, neutral, autonomous and purified concepts, and we leave apparent the history of construction of concepts and the links with the situations (problems) that motivated them, we begin to notice certain identities in fields of knowledge usually considered disjoint. For example, we can see identities between the construction of a writing system and the construction of mathematics.

We now pass to the educative proposals of Paulo Freire and its connections with mathematics.

PAULO FREIRE AND MATHEMATICS

Paulo Freire (1921-1997) was a Brazilian educator very attentive to the understanding of human affairs and the overcoming of oppression situations. His work had focused on the problem of adult illiteracy, which in Brazil is still an alarming problem: around 14 million illiterate adults (UNESCO, 2014). At first, his educational practices involved a direct contact with popular groups, to discourse and present concepts. But, in the early 1960s, he realized that the abstract speech filled with stabilized concepts did not cause in the oppressed/oppressor a different attitude toward situations of oppression. That practice failed to stimulate in each person the awareness about his own social role. Thus, he reversed his educational action, he began to look for ways to stimulate in each person and group the expression of their own word:

> If you can reach a high level of discussion with popular groups, regardless of whether or not they are literate, why not doing the same in a literacy experience? Why not critically engage learners in assembling their graphics system of signs, as subjects of this assembly and not as objects of it?" (Freire & Betto, 1985, p.15).

Keeping in mind a conception of mathematics that is not disconnected of the things of the world, Paulo Freire had a very mathematical way to understand and teach literacy. He started from the events of life to construct concepts. Both in his speeches as in his writings we see plenty of stories, lived cases, personal narratives and descriptions. All these contributed to bring to the reader the perception of the experience lived by him, or by the group. The formulation of concepts would come directly from these situations. Local issues were the main matter: this is a situated approach.

In Brazil, around the 50's, literacy was done by booklets ("cartilhas", in portuguese) that proposed the exhaustive repetition of artificially assembled phrases. For example, to teach the letter "P", "Pato: pa, pe, pi, po, pu" (Duck: da, de, di, do, du). Literacy was taught separately from any contextualization, just as a combination of signs and repetition of sounds. This method left behind a history of illiteracy, naturalizing the view that reading and writing would be very difficult things.

The method of Paulo Freire had as starting point the person's reflection on his own social condition, what he called "awareness". His proposal was to consider the speech of each one about his own life to construct the "vocabulary universe" from which he would extract the words and phrases to use in literacy. For each person or group, a new vocabulary universe. In this way, the person himself was involved in the construction of his learning process. Keeping apparent the construction of the system of writing, he sought to make evident the links of abstract language (the "graphics system of signs", in his words) and the things of life. This was a very successful method, recognized and welcomed by the Brazilian government. But then came the military coup of 1964. Under the eyes of the repressive government, Freire's approach was considered subversive and so, it was interrupted. Freire spent 75 days in prison accused of subversive and ignorant, and then was sent into exile.

We bring here part of a recent speech by Paulo Freire in 1996, a year before his death. We would like to stress here a great affinity with the mathematical way of thinking. For this, we go back to the discussions about the foundations of mathematics of the beginning of the twentieth century. Recall Russell's paradox, a self-referenced property with a negation. The paradox arises when this property is applied to an expression of itself. Recall Gödel Theorems, the evidence of incompleteness, whose proof goes in the same kind of reasoning, and recall the completation hierarchy proposed by Turing, that follows directly from the perception of incompleteness. An infinite search for completeness, a step-by-step process, each step more complete than the previous but never reaching completeness. Both the construction of Russell's type theory to deviate from contradiction, and the construction of Turing's hierarchy to deviate from incompleteness, employ a constructive loop that never reaches its objective. This way of reasoning is also present in the speech of Paulo Freire.

> (…) educational practice is not founded solely on the ontological inconclusiveness of the human being, but on a conscious awareness of this inconclusiveness. Education is founded on these two feet, one the inconclusiveness, the other the conscious awareness of this inconclusiveness. Human educability has no explanation other than the assumption of aware inconclusiveness. In the same way, these are the pillars which support hope. Can you imagine how incongruous it would be to be inconclusive, as we are, and conscious of this inconclusiveness; not to be immersed in a permanent movement of search, of quest? The being which does not search is the one that, being inconclusive, does not know of its inconclusiveness. Here is an example: the tree which I have in the garden of my house is also inconclusive, since the phenomenon of inconclusiveness is a vital phenomenon, it is not exclusive to human beings. But the level of inconclusiveness of this tree has nothing to do with my level of inconclusiveness It is inconclusive, in the same way that my dog is inconclusive, but dogs do not know themselves as inconclusive. In our case, we assumed the inconclusiveness and in assuming it, we are led to search. (Freire, D"Ambrosio & Mendonça, 1997,p.9)

Russell's paradox, as well as the proof of Gödel theorem, employs a negated self-referenced sentence: a predicate that cannot be applied to itself, or in Gödel theorem, a sentence that asserts its own unprovability. Freire's argument is about the conscious of its own inconclusiveness. It unfolds through a negated property (non-conclusiveness) and a reflection (conscious awareness of non-conclusiveness). From this combination of elements, Freire points out what would be an incongruence: to be inconclusive, and conscious of inconclusiveness, and not to be immersed in a permanent movement of search. From the perception of this incongruence results the search movement, a search that comes from inconclusiveness, just as the perception of incompleteness induces the search for completeness.

The argument of Paulo Freire presents a coincidence of elements, and an approximation to the way of thinking of mathematicians. A very similar way of reasoning both in mathematics and in education. This may seem odd to those who consider Gödel theorem in its expression in the jargon of mathematics of the 1930s. When an idea is expressed in the jargon of a discipline, it acquires a purified appearance, as if it belonged exclusively to the domain of that discipline. Then we forget that it came from life, from the experience of someone, from someone's way of deciphering the world, and took a form in a discipline: in mathematics, or in education.

FOR A SITUATED APPROACH TO MATHEMATICS

> Peasant: I see now that there is no world without man.
> Educator: Let us think, absurdly, that all men in the world died, but stood the land, stood the trees, the birds, the animals, the rivers, the sea, the stars. Would not be all this the world?
> Peasant: No! It would lack those to say: This is the world!
> (Freire, 1968, p.41)

Over nearly five decades thinking and rethinking education, Paulo Freire's work shows the constant attention to humans in their world, working and thinking about the issues in their place and their time. Freire's educational proposals focused on the learning process, which cannot be separated from the learner and his world. The many stories that Freire tells in his texts make apparent the steps that he travels in building his concepts, the links between abstract concepts and life world.

But the national and global recognition of Paulo Freire as a great thinker in the field of pedagogy multiplied references and citations to his work. On the way of modern scientific practices, knowledge acquires major importance to the extent that hides its links to worldly things. Thus, we see now the work of Paulo Freire reported in a purified tone, neutral and universal, which leaves no room for the inspiring stories. Strengthened by scientific legitimation mechanisms, but weakened in their nesting in the world, the work of Paulo Freire becomes a jargon. This raises the various criticisms and misunderstandings.

This is the same process of distancing from the world that the hegemonic mathematics has suffered along its journey in the modern era. The distanced narrative, free from ties to the worldly things, is common in mathematical texts produced in the modern era and legitimized by the scientific community. A body of knowledge that, once stabilized, get rid of its links with the issues it was intended to solve. It comes into existence as autonomous formulations, ahistorical entities. From this comes the difficult math: when it is not clear the connection between its constructs and individual resources that allow them to acquire significance. This mechanism serves to a certain configuration of power, making mathematics a subject for a few, as commented Paulo Freire:

> In my generation of Brazilians from the Northeast, when we referred to mathematics, we were referring to something for gods or for geniuses. There was a concession for the genius individual who might do mathematics without being a god. As a consequence, how many critical intelligences, how much curiosity, how many enquirers, how many abstract capacities in order to become concrete, have we lost? (Freire, D"Ambrosio, & Mendonça, ,1997,p.8)

Looking at the case studies in communities, we can collect evidence that people build their mathematical abstractions as answers to their needs. This weakens the foundations of a universal body, unquestionable and untouchable knowledge. This latter sets the mathematics of the gods, completely disconnected from things in life, such as the mathematics that was taught to Paulo Freire when he was a boy in the thirties. It is the understanding of such an abstract construction that seems to require a special kind of talent, something of genius.

At the same time, “the language God used to write the world” settles the role of hegemonic mathematics as a reference of the correct mathematics. This makes impossible to recognize, also as math, other constructions of life. In these constructions, numbers, measures, among other elements of the hegemonic mathematics, are rarely shown. As they have a different historical process, these constructions may show other foundational elements. Thus, a situated approach, that is, an approach that makes clear the place, time, cultural elements and does not separate the construction from who conceived it, is necessary to recognize this mathematics of life.

A situated approach leads to an understanding and practice of mathematics in such a way intertwined with other skills that is no longer possible to establish precise boundaries for mathematics. It takes into account a conception of mathematics as the skill of inventing concepts to propose solutions to the demands of life. This requires both the expression of these demands in a specific jargon as the design of mechanisms to operate these concepts so that they can follow the changing of things of life. It requires a constant adherence to life situations, a seam between concrete and abstract to conceive solutions. This adherence does not exclude the learning of hegemonic concepts, but considers the translation of these concepts to shape life problems. But this shall be done in accordance with the settlement of these concepts in local situations so that they acquire new links with the world and therefore new possibilities of comprehension.

A situated approach does not mean to understand the "mathematical rationality" of a collective, or to build models to explain other cultures, because this would require to take one's own rationality as a cultural reference. A situated approach means to consider the solutions of a collective from their own problems, close to their lives and their demands. Then you may think in Mathematics in a broad way, requiring and cooperating with knowledge of any kind and with the everyday life.

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