

Paulo Freire, Mathematics and Policies that Shape Mathematics

Isabel Cafezeiro^{1,2}

• Ricardo Kubrusly²

• Ivan da Costa Marques²

• Edwaldo Cafezeiro³

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Abstract

Purpose This paper proposes a situated understanding of mathematics, which means recognizing mathematics as locally and collectively constructed knowledge, in opposition to the universalist and neutralist conceptions of mathematics. We consider proposals formulated by Brazilian intellectuals of the 1920s and 1950s, as well as the political and social conjuncture of contemporary Brazil.

Methodology We start section “[An Act of Vandalism](#)” in a critical position regarding the current Brazilian social and political conjuncture. We show that this has been provoking the strengthening of education policies that are far from reflection and dialogue (“[The Brazilian Common National Base Curriculum](#)” section). In a counterpoint to these policies, we consider the ideas formulated by the Brazilian educator Paulo Freire from the 1950s onwards (“[The Reversal of an Authoritarian Scenario](#)” section), as well as the proposals formulated by the Brazilian intellectuals of the 1920s who founded the “anthropophagic movement”. They argued in favour of a Brazilian translation and the appropriation of foreign knowledge in both the artistic and intellectual fields (“[Anthropophagic Mathematics](#)” section). We also consider the historical course of the construction of hegemonic mathematics to show a process of untying mathematical knowledge from the demands of life to constitute an abstract, neutral, universal and purified body (“[Formal \(Deductive\) and Informal \(Procedural\) Mathematics, Both Social Constructions](#)” section).

Result and Conclusion Starting from these reflections and examples in the Brazilian scenario, we verified possibilities of constructions of local mathematics from the recognition of the social experience of mathematics. This opens the space for the development of mathematical proposals that best meet the demands of each locality, be it in Brazil or in India.

Keywords Sociology of mathematics Paulo Freire Education in Brazil

Isabel Cafezeiro isabel@hcte.ufrj.br

Ricardo Kubrusly risk@hcte.ufrj.br

Ivan da Costa Marques imarques@ufrj.br

Edwaldo Cafezeiro cafezeiro@uol.com.br

¹ Computer Science Institute of Universidade Federal Fluminense (IC/UFF), Niterói, Rio de Janeiro, Brazil

² Graduate Program of History of Science and Techniques and Epistemology of the Federal University of Rio de Janeiro (HCTE/UFRJ), Rio de Janeiro, Brazil

³ Faculty of Letters of the Federal University of Rio de Janeiro (UFRJ), Rio de Janeiro, Brazil

An Act of Vandalism

Our starting point is the letter (Freire 2016) sent by the widow of the Brazilian educator Paulo Freire, Mrs. Ana Maria Araujo Freire, to the current President of Brazil Michel Temer in June 2016.

It is worth clarifying: Michel Temer was newly sworn in, on 13 May 2016, replacing the elected president Dilma Rousseff. Under a powerful appeal of the press, President Dilma was accused of delaying the transfer of funds to the payment of social programmes. Some of these programmes were started under the previous government, of President Lula, in order to interrupt the state of absolute poverty of part of the population, and others began in Dilma's government to provide housing to the impoverished population. The delay in the transfer of funds is a practice that had been adopted by the sequence of previous governments that did not prioritize social programmes (Cardoso 2016a). But this gained the status of crime in Dilma's management, justifying the impeachment. The installation of the new president set up a coup, since extremely serious acts intentionally impairing the constitution were not found in Dilma's management (Cardoso 2016b). This episode made Brazil relive moments of a time already considered a thing of the past: where there is an invitation to the resurgence of with authoritarian and retrogressive proposals, especially in the field of education.

In life, Paulo Freire (1921–1997) had faced another coup in 1964. He was a Brazilian educator substantially interested in understanding human issues and overcoming situations of oppression. He was also extremely troubled with the Brazilian reality, a country dominated by illiteracy. At first, his educational practices involved a direct contact with popular groups, to discuss and present concepts. But, in the early 1960s, he realized that the abstract speech filled with stabilized concepts did not cause in the oppressed/oppressor a different attitude towards situations of oppression. That practice failed to stimulate in each person the awareness about his own social role. Thus, he reversed his educational action. He began to look for ways to stimulate in each person and group in the expression of their own word. He devised an approach that caused changes in the literacy scenario in north-eastern Brazil, a region challenged by drought and poverty. The success of Paulo Freire's method was known and accepted by the Brazilian government of that time, under the presidency of Joao Goulart. But then came the military coup of 1964. Under the eyes of the new repressive government, Freire's approach was considered subversive and so was interrupted. Freire spent 75 days in prison accused of subversion and ignorance and was sent into exile (Gadotti 2015). Brazil had entered into dictatorship, authoritarianism, violence and repression of the most basic freedoms. This lasted for twenty years.

We shall present some considerations about the mathematical character of Freire's approach. But before, let us return to the letter of the widow and the legal successor of Paulo Freire, to understand the current situation of education in Brazil and the mathematics that was taking shape at this juncture.

Ana Maria Freire addressed the current substitute president of Brazil to protest the act "that came from within the government through SERPRO network. This is a public body and therefore under the responsibility of the Brazilian State" (Freire 2016). The act that mobilized Ana Maria was the change in the content of Freire's biography in the free encyclopaedia Wikipedia. The inserted text identifies Freire's work as an educational project "outdated and of weak character, Marxist doctrinaire and manipulative".

It is worth emphasizing that Brazil is living a time of strengthening of authoritarianism and intolerance. This scene includes street demonstrations in favour of the return to dictatorship by the middle and upper classes who find themselves threatened by the recent access of the lower classes to the minimum conditions of dignity and citizenship. This also includes the movements of rural benches of agribusiness elites in favour of the liberation of the use of weapons, the "bullet bench", and also the various homophobic and racist manifestations.

In the field of education, these demonstrations are taking shape from movements that defend what they say is a "neutral education" and fight what they say is an "ideological indoctrination" and so reject any critical and reflective education in the position of being a mere transmitter of content. Currently, several projects to institutionalize these proposals as national law are ongoing in the legal Brazilian proceedings (BRASIL 2015).

It is clear here that Paulo Freire, an encourager of social awareness and its transformative purpose, is a prime enemy of these movements. In a return to the old medieval inquisition practice, these groups put

Freire's books in a list of what they consider "dangerous and prohibited". Their eyes are closed to the national and international recognition of Freire's ideas in the field of pedagogy.

In this confusing scenario, Brazil continues the project that the government considers essential for education: the establishment of a unified curriculum for the whole territory. The Common National Base Curriculum (CNBC 2016) sets a minimum list of contents and shows an educational policy strongly committed to a universal standard of quality, regardless of local needs. The list of contents acts as a way to control what is taught in every school, seeking to ensure that things are moving in the "right direction" towards the success of the developed world. What certainly stands out in this strategy are the local interests, respect for culture and regional singularities, although these are precisely the priorities listed in the text.

This reflects an educational police that looks straight to the world but turns its back to Brazilian people.

The Brazilian Common National Base Curriculum

We give a very brief idea of how the mathematical part of the Common National Base Curriculum is organized. First of all, let us stress that the government educational proposal starts with a rigid separation between what is considered to be the "technical part", that is, the curriculum organization and the conditions where this is supposed to work. This last point is to be considered elsewhere, in proposals that focus only on the social. Thus, in this document we will not find any consideration about the school conditions, remuneration of teachers or about the miserable conditions of students, so let us suppose an idealized Brazil, where children are not hungry, teachers are happy and willing to achieve goals, and schools are in a condition to receive students.

In early childhood education (under six years old), there is not yet an explicit separation between disciplines. So the goals are described in five experimental fields: (1) the self, the other and the us, (2) body, gestures and movements, (3) listening, speech, thought and imagination, (4) traces, sounds, colours and images, and (5) spaces, times, quantities, relationships and transformations. We see here the possibilities of the development of mathematical knowledge in all these fields, with emphasis on the fifth. The proposal shows freedom for consideration of local skills from the particular circumstances of each corner of Brazil. This is a situated approach to mathematics, that is, an approach that is built and justified according to the local reality.

For the next 12 years of schooling, which begins around age six, mathematics is mentioned as a self-contained topic, a discipline with well-defined borders, separated from the others. From this part, the mathematical proposal is organized in to five main divisions that run through the entire curriculum in an increasing degree of abstraction and complexity: (1) geometry, (2) quantities and measures, (3) statistics and probability, (4) numbers and operations and (5) algebra and Functions. Step by step, mathematics will be separated from the local reality and becoming an essentially abstract, universal and neutral body of knowledge. Before the presentation of each year of schooling, there are some introductory words stressing "the necessary approximation of the mathematical knowledge and the world of culture, contextualization and critical instrumentation, as principles which are the starting point for teaching practice", as well as the importance of dialogue with other disciplines. But these indications are lost throughout the presentation, year after year. What we see is a list of contents, following the organization of hegemonic mathematics, and ignoring the applicability of concepts to the wide Brazilian regional diversity and the meaning that those abstract concepts that may motivate the learner. The proposal suggests a mathematics consolidated in formulas and applications, neglecting to present the course of formation of this knowledge. This configures a mathematical approach as a naturalized and unquestionable knowledge and therefore authoritative and averse to host local expressions. We quote here some examples to show how the approach of mathematics is abstract and disconnected from life and how the content list deviates from the principles addressed as main educational points. Among other topics, we can see in the first year: "Composing and decomposing numbers at least 30 (e.g. $10 = 2 + 8$, or $10 = 5 + 5$ or $10 = 1 + 9$ or $10 = 11 - 1$; $17 = 10 + 7$ or $17 = 12 + 5$)". In the sixth year, "Solving and elaborating problems involving equations of 1st degree of the type $ax + b = c$, the set of natural numbers, through trial and the principle of equality". And in the ninth year: "Developing binomial products as $(x \pm y)^2$, $(x + y)(x - y)$, $(x + a)(x + b)$, describing a process for obtaining the result". Thus, there is nothing new in this proposal besides a rearranging of contents in schooling years. There is nothing guided to improve the

learning, nor to search for a mathematical dialogue that would improve the local creative capabilities to deal with mathematics. There is nothing concerning the difficulties of Brazilian children to deal with this formal knowledge. Learning mathematics will continue to be something for few in Brazil even after the Common National Base Curriculum, exactly in the same way as it was at the time that Paulo Freire was still a student.

In my generation of Brazilians from the Northeast, when we referred to mathematics, we were referring to something for gods or for geniuses. There was a concession for the genius individual who might do mathematics without being a god. As a consequence, how many critical intelligences, how much curiosity, how many enquirers, how many abstract capacities in order to become concrete, have we lost? (Freire et al. 1997, p. 8)

The Reversal of an Authoritarian Scenario

Here again, we return to Freire. In the opposite direction of an educational proposal fixed in stabilized contents, the practice of Paulo Freire was based on dialogue and flexibility, in building concepts from the location, the individual and his social life. His proposals were the fruit of experience. This is clearly perceived in his books by plenty of stories narrated involving several groups: natives, peasants, workers, students, teachers, prison inmates (Freire and Betto 1985; Freire 1987). The observation of life and living was not for him only theoretical proposals. This was his own way of operating. He went to the field and made policy. In the period, he assumed the office of education of the city of São Paulo; in 1989, he widely criticized the traditional pedagogical approach, regulated by contents. This was, for him, a repetition of pedagogy that disconnected social context and in which teachers have the memorized mastery of concepts they teach, but find it difficult to work without a fixed model, in the flow of local things:

I think that a thought is valid only when it is possible to be redone. When a thought or theoretical proposal has to be repeated strictly as it was made I do not believe in it. I think that the validity of the proposal is the possibility that it has to be re-created, to be reinvented. (Freire 1989)

Therein lies the point of conflict: the institutionalization of an educational project focused on fixed contents to cover the whole nation, chokes the political dimension of learning and disregards the local identities. This opened a receptive space to approaches based on repetition of concepts.

In fact, on 31 May 2016, the project of the Common National Base Curriculum was brought to the government's legislative body (BRASIL 2016). Discussions were led by enthusiasts of conservative movements focused on fighting a supposed "ideological hegemony of the left". Conservative groups (ESP 2016) were strengthened and they increased the attacks on the figure of Paulo Freire and his proposals, as denounced by the widow Ana Maria Freire.

It is a messy scenario because, at the same time, these conservative movements both welcome the government's proposal and also criticize it because of their humanistic content. With respect to the humanities, the conflicting point is that the government proposal includes knowledge of and respect for minority cultures and gender issues. But, with respect to the mathematical part, there are no criticisms. Why?

We now turn to the mathematics of Paulo Freire.

Freire had a very mathematical approach to literacy, as he used to say, "an assembly of a system of signs". He sought the construction of concepts drawn from the reflection on the everyday events. As we said, Freire's proposal shows an abundance of narrated cases, descriptions where one can see his effort to bring to the reader the perception of the experience lived by him. From this perception, Paulo Freire would encourage the formulation of concepts.

At that time, literacy was done from the exhaustive repetition of sentences artificially assembled, for example, "Vovo viu a uva" (Grandma saw the grape), a repetitions of sounds in V to teach the letter V. This was a formal system of signs disconnected from the everyday life and uniquely based on the

repetition. These formal combinations of signs did not communicate anything to do with the present, to the worker or to adult learners, not even to the learning children. This method left a trail of illiteracy, naturalizing the view that reading and writing would be very difficult things.

This scenario was not different from the mathematics that was taught then and that is still taught nowadays in schools in Brazil, an airtight knowledge, trying to appear strictly formal, and with no ties to the world of the learner. This is a mathematical statement based on stabilized concepts and application of ready-made formulas where the apprentice, helpless and submissive, has the role of absorbing and repeating.

Paulo Freire proposed a different method to literacy that had as its starting point the individual's reflection about his own social condition, which he called "awareness". Thus, the individual himself was involved and committed in the construction of his learning process.

From the speech of each one about his own life, Paulo Freire withdrew the words and phrases to be used in the construction of the writing system. That is, keeping visible the process of construction of the system of writing he sought to make evident the bonds of abstract language with the things of life. Even more, he repositioned the role of the learner, which became an active agent in the construction of their own knowledge. The importance of this approach can be noted in a news article published in 2013 to commemorate 50 years of the first Paulo Freire literacy group. Only in that group, in the small city of Angicos, 300 workers were literate in 40 h:

The 83-year-old Idália Marrocos da Silva says she remembers 'like it was today'. 'We were going to a house and we had class in the room. That time these classes happened everywhere: in the church, in the police station, in people's homes. A lot of people learned to read with these classes', she recalls. Easy smile and good memory, Mrs Ida'lia remembers that many people were afraid to go to the classes because at the time people said that Paulo Freire was a communist and that the students would be persecuted. 'A lot of people were afraid. My mother did not want me to go, but these classes mobilized the entire city. It was almost a revolution and I wanted to be part of', she says, in the rocking chair in a simple house where she lives alone. (Zauli 2013)

Two decades later, as Secretary of Education of São Paulo, Freire situated the teaching of mathematical knowledge exactly in the same way as he considered the teaching of reading and writing in the 1960s:

The process of knowledge has an individual moment, but it is a social process. Dialogue is an epistemological requirement, is part of the knowledge process. Is the dialogue possible in the exact sciences education? People say 'no, it is not'. In this way, they reduce the teaching of mathematics to a mere transmission: repeat and cause the mechanical memorization of the profile of the concept that they describe. The dialogue in mathematics is the action of inquiring about the relationship between the learning of mathematics (arithmetic, algebra) and social experience. By the time the student learns the rigor that mathematics suggests he should be open to a relationship or an understanding of social connected to mathematics. (Freire 1989)

From here, then we understand why the current movements that stand opposed to critical education are not opposed to the mathematical part of the Common National Base Curriculum. It is that, even today, in Brazil, we did not learn how to deal with mathematics in a relationship of dialogue, from the social experience. We do not have intimacy with the mathematical entities to the point of reliving them, try them and transform them. We are mere repeaters. Thus, the discussion regarding mathematics in the Common National Base Curriculum is restricted to a rearrangement of content in school years, something that conservative groups welcome.

Anthropophagic Mathematics

Now we focus on the question raised by Paulo Freire: Is dialogue possible in the education of the exact sciences? In order to approach mathematics as a social experience, we propose a double strategy. The first is to consider the historical route of the construction of mathematics. This is essential to understand mathematics as a social construction, to make clear that its entities and concepts were conceived as a response to a particular situation attending a certain configuration of power that took place somewhere, in some time. As soon as one realizes that mathematics has always been built as a social experience, one also happens to think of the possibility of a mathematics that is able to meet the demands of our reality in our time, our social experience, an anthropophagic Mathematics.

This is not a denial of hegemonic mathematics. This is the translation of a mathematics that has been traditionally imposed in order to best meet our needs. In the 1920s, a group of Brazilian artists and intellectuals revolted against foreign domination in its various forms, but mainly the domination of thought, language and artistic production. They created an anthropophagic journal and founded the anthropophagic movement (Andrade 1928). Referring to the native culture Tupi-Guarani, the movement popularized the parody “Tupi or not Tupi, that is the question” (written as it is here, in English) claiming the need to rethink the national identity in the face of foreign imposition. The anthropophagic movement was inspired by the cannibalistic practice of certain Brazilian tribes to eat the enemy captured in wars understanding that in this way their qualities would be acquired. Thus, the anthropophagic movement proposed to eat the foreign, swallow it, feed on what serves Brazil, and vomit what does not serve. This mix of foreign ideas with Brazilian ideas makes it possible to produce something new, a “Brazilian new”. Abaporu (aba = man; poru = eating), authored by Tarcila do Amaral, was the beautiful screen that inspired the movement. About language, they demanded the freedom from formal rules, to write the spoken language of Brazil, without the shackles of Portuguese grammar: “Pronouns? I write Brazilian” (Andrade 1987). About logic, the movement claimed “We never admitted the birth of logic among us”.

In order to recognize our logic, our mathematical language, our mathematical way of living and open space for mathematical translations according to our demands, we consider in the next two sections, the understanding of mathematics as a social construction (“[Formal \(deductive\) and informal \(procedural\) mathematics, both social constructions](#)” section) and the possibilities of recognizing local mathematical expressions (“[Possibilities of local mathematics](#)” section).

Formal (Deductive) and Informal (Procedural) Mathematics, Both Social Constructions

We will argue that mathematical concepts and entities take shape from the circumstantial conditions of the place and time they were set out, as a demand of social experience. Our goal is to undo the conviction that the mathematical entities and concepts are supported by themselves, independently of life and its policies. Contrary to this, we show that mathematics is socially constructed. This puts in check a number of conceptions that hinder the recognition of our mathematics, for example, the role of the hegemonic mathematics as the unique and universal form of mathematics, and all the authoritarianism that this imposes on us.

We take as a running example the process of the construction of formal deductive mathematics as a standard of “real mathematics”. We will show the situations of life that caused the placement of this form of mathematics as the hegemonic one. At the same time, these situations also placed any form of non-deductive, non-formal mathematics as a subjective and depreciated mathematics. This process that took place since ancient Greek mathematics and has lasted until the mid-twentieth century, despite the recognition by some mathematicians that the hegemony of deductive mathematics is not sustained by mathematics itself: “Proofs which rely on informal methods have, in their favour, all the evidence accumulated in favour of Church’s Thesis” (Rogers 1967, p. 21).

Mathematics is born indistinct from art, inseparable from life. It is the expression of men and women in their demands, their conflicts, in the pursuit of building their identity. Thus, as an expression of things in life, mathematics continues throughout human history. When leaving the cave, the primitive human is faced with the unknown and the unexpected: a bison. Frightened, he returns to the cave and draws the bison. He also draws himself, printing his own hands on the walls. He draws himself facing the bison. Thus, by drawing, he recognizes himself and he rethinks his fears and creates ways of transforming the world in which he lives. The primary issue of being human is, and always has been, outsmarting death, so

he leaves marks that should outlast his own life. Being aware of his own death, the human looks for ways to extend his life indefinitely. It is the search for the infinite. There originates all the mathematics we know today: in the moment that the human, by representation, seeks to understand time and space and thus builds its place as subject to himself and of his world. “Mathematics as a condition of being in the world”, said Paulo Freire in recent times (Freire 1987).

The closeness between the representation and the lived problem motivates a procedural form, a description of “how to”. The drawings on the walls, that is, the mathematics of those times, show the ways to hunt, ways of living in a community. This was the science of that time.

Other mathematics emerged in other times from other demands, always in response to immediate needs of life. The human being, fixed in the land by agriculture around 9000 BC, began to feel the need to work in groups, to divide the production and to exchange. There arose the fairs, villages, first cities around the Tigris and Euphrates rivers. Accompanying this was another way of being in the world, where there emerged other ways to express one’s own existence.

The incipient trade generated the need to register the inflow and outflow of goods, hence the symbols minted in clay tablets that gave origin to the cuneiform script of the Sumerians (3500 BC). These plates survived the destruction of Sumer by wars and came to the Babylonians (1830 BC to 539 BC), making available a knowledge that possibly had already reached what we now call fractions, algebra, quadratic and cubic equations and something that later received the name of “Pythagorean Theorem”. Around the same time, the Egyptians invented sacred symbols, their hieroglyphics to write messages in temples and tombs. Once again, mathematics and life appear indistinct, as expressions of being in the world.

The Sumerian people, and later, the Egyptians and Babylonians (between the XVIII and VI BC), had improved the descriptions of their processes because of recurrent needs to measure areas. Much later, around the year 440 BC, Herodotus, a great scholar of the world and very attentive to the way of life of people before his time, took care to record the customs of ancient peoples in the region around the Mediterranean Sea and North Africa. He made clear the demands from climate variations and flooding of rivers in times preceding his birth. He showed in his reports that in the case of river islands, and land on river banks, not even the ground on which one steps is consolidated:

Sesostris[king of Egypt and Ethiopia] also, they declared, made a division of the soil of Egypt among the inhabitants, assigning square plots of ground of equal size to all, and obtaining his chief revenue from the rent which the holders were required to pay him year by year. If the river carried away any portion of a man’s lot, he appeared before the king, and related what had happened; upon which the king sent persons to examine, and determine by measurement the exact extent of the loss; and thenceforth only such a rent was demanded of him as was proportionate to the reduced size of his land. From this practice, I think, geometry first came to be known in Egypt, whence it passed into Greece. The sun-dial, however, and the gnomon with the division of the day into twelve parts, were received by the Greeks from the Babylonians. (Herodotus, 440 BC, Book II, paragraph CIX)

Assuming a different form in every flood, the variant ground seems to have demanded a lot of mathematics. Possibly from this, the people developed the reading of the stars to understand the movements of the rivers, the calculation of areas to redivide the land and recalculate the tariffs on leased land to the people by the Pharaoh. Thus, we see that here, as in the caves, adherence to a problem gives rise to a procedural style, emphasizing “how to”, a geometry: how to measure the earth.

There is much evidence that mathematical knowledge did not originate in the noblest intellect through logical reasoning. On the contrary, these ancient stories show that it was born at the fair, in trade, agriculture, faith, to solve immediate issues of life and in the human search for explanations in face of his finitude.

By the sixth century BC, the Greek people started to take notice of the Egyptian and Babylonian mathematics. But this was a society marked by a tradition of assemblies that induced a meticulous arrangement in arguments to become convincing. This practice seems to have penetrated the

mathematical texts. The extreme care with the form of the statements caused in the mathematical presentation a certain appearance of detachment from the problems that have served as inspiration. Mathematics was gradually appearing to be something exclusively from the intellect. There prevailed an elaborate presentation, polished, prioritizing the linear chaining, a deductive mathematics, which happened to be confused with the mathematical way of thinking. Long after the Greeks, Bourbaki would exalt the deductive way of presenting mathematics as the path to discovery in mathematics: “[The axiomatic method] is not a new invention; but its systematic use as a discovery tool is one of the unique features of contemporary mathematics” (Bourbaki 2006, p. E.I.8).

One of the most important and oldest mathematics books is *The Elements*, of Euclid of Alexandria, written around 300 BC. According to the translations that are available today (Euclides 2009), the text of Euclid starts directly into a sequence of definitions that fix the meaning of some basic concepts. There is neither an introduction nor an explanatory word. It is assumed that the definitions, as well as what comes next in the other 13 books that make up *The Elements*, should be clear and obvious, so there is not need for an explanation. Following the definitions, there is a sequence of common concepts, that is, statements that would be evident in any field. Then, the postulates, statements that would be evident in the mathematical field and finally the propositions, should be obtained by proofs, directly from the definitions, common notions and postulates.

This organization seems to reflect the concept of deductive science formulated earlier by Aristotle, who sought in arithmetic and geometry the terms that would become his own philosophical terms. This served him as example of thinking required by the philosopher. In the second part of the *Organon* (the name of his works about the logic) entitled *Prior Analytics*, he explained what he meant by a deductive (demonstrative) science:

We must first state the subject of our inquiry and the faculty to which it belongs: its subject is demonstration and the faculty that carries it out demonstrative science. We must next define a premiss, a term, and a syllogism, and the nature of a perfect and of an imperfect syllogism; and after that, the inclusion or noninclusion of one term in another as in a whole, and what we mean by predicating one term of all, or none, of another. (Aristotle, p. 81)

Thus, Aristotle claimed the schematic way for philosophy, to incorporate the accuracy and rigor that he saw in mathematical presentations. His text, however, did not exhibit these characteristics directly. Despite the use of letters to represent terms, his writings are still full of explanations, examples, assumptions, in a completely different style from the text that Euclid presented in *The Elements*, which brought to mathematics the polished forms revered by the philosopher as a clear, dry and clean thinking.

For some historians of mathematics (Boyer 1991, p. 71), what is new in *The Elements* regarding the mathematics of the time are not exactly the mathematical results, but the ability to organize and report information. They consider that Euclid excelled as a teacher of mathematics, not as mathematician as he did not present new results¹. Hence, there would have been an explicit investment on the part of Euclid in the sense of clearing his arguments, ridding them of their empirical character (see footnote 1). This point is important because it shows a constant concern to emphasize a supposed superiority of deductive mathematics. As has been said here, later, in the mid twentieth century, the deductive way of presenting mathematics came to be identified as the way of thinking of a mathematician. This suggests a contrast between the “logical reason” of mathematicians and the chaotic way of thinking of people in general, positioning the former as intellectually privileged. Morris Kline made clear that

Euclidean geometry did not come into being in this deductive manner. It took three hundred years, the period from Thales to Euclid, of exploration,

¹Another author, in criticizing the myth of superiority of formal mathematics and the Greek primacy in the discovery of such mathematics, presents strong evidence that there has never been a Greek named Euclid. He argues that the emergence of this fictional character arose from the interests of the Crusading historians, and was later welcomed, meeting the interests of the construction of the modern historiography of Western mathematics. According to Raju, the proofs in *The Elements* are essentially empirical (nondeductive). However, there was a certain convenience on the part of the mathematical philosophers of the twentieth century in convincing that Euclid failed in his intention of deductive proof. Raju's arguments can be found on his webpage (<http://ckraju.net>), especially in the paper “Education and the church: decolonizing the hard sciences”.

fumbling, vague and even incorrect arguments before the Elements could be organized. Thus the Elements is the finished and relatively sophisticated product of much cruder, intuitive thinking. Even this structure, intended to be strictly logical, rests heavily on intuitive arguments, pointless and even meaningless definitions and inadequate proofs, as the nineteenth century mathematicians realized. What is most relevant, however, is that this deductive system came about after the understanding of all that went into it was achieved. (Kline 1976)

For Archimedes (287 BC to 212 BC), this separation between a mathematics of life and another, an abstract mathematics, seemed as clear as the dependence between these two presentations of mathematics. That is, he perceived the impossibility of a pure form for mathematics, whatever the way. Archimedes was a mathematician whose work showed a clear adherence to life. He started from mechanical experiments to build his hypotheses, and from this, he built geometric proofs. However, we now know that he realized the need to express his mathematics in deductive terms. In the year 1906, the Danish philologist Heiberg had access to a scroll of ancient Greece, whose contents had been erased and overwritten with liturgical texts, which was a common practice in the Middle Ages. Heiberg deciphered the original writings and identified Archimedes' texts. Among them is a letter to Eratosthenes, in which Archimedes recognized the need for a deductive version for his results and delegated this task to Eratosthenes. In this letter, he explains his way of thinking mathematically, stressing the importance of reasoning about mathematics through things of life (mechanics):

Some time ago I sent you some theorems I had discovered, writing down only the propositions because I wished you to find their demonstrations which had not been given. (...) I have thought it well to analyze and lay down for you in this same book a peculiar method by means of which it will be possible for you to derive instruction as to how certain mathematical questions may be investigated by means of mechanics. And I am convinced that this is equally profitable in demonstrating a proposition itself; for much that was made evident to me through the medium of mechanics was later proved by means of geometry because the treatment by the former method had not yet been established by way of a demonstration. For of course it is easier to establish a proof if one has in this way previously obtained a conception of the questions, than for him to seek it without such a preliminary notion. (Archimedes, 1909, p. 9–10)

Archimedes refers to Eudoxos mentioning a previous situation where the conception of a deductive proof hid the perception of the concept by Democritus. But, identifying his own situation with the episode Eudoxos and Democritus, he drew attention to the essentiality of the work of conceiving concepts on the mathematics of life. He did not give up his role as a mathematician, a concept building:

Thus in the familiar propositions the demonstrations of which Eudoxos was the first to discover, namely that a cone and a pyramid are one third the size of that cylinder and prism respectively that have the same base and altitude, no little credit is due to Democritus who was the first to make that statement about these bodies without any demonstration. But we are in a position to have found the present proposition in the same way as the earlier one; and I have decided to write down and make known the method partly because we have already talked about it heretofore and so no one would think that we were spreading abroad idle talk, and partly in the conviction that by this means we are obtaining no slight advantage for mathematics, for indeed I assume that some one among the investigators of to-day or in the future will discover by the method here set forth still other propositions which have not yet occurred to us. (Archimedes, 1909, p. 10–11)

We see that the ancient mathematics suffered a lapidary process prioritizing the thread of the argument. This process positioned deductive mathematics as a privileged way of thinking, gradually omitting features of space–time of the original problem. Moreover, mathematics was also mingling with its own

presentation. The deductive form has come to mean the mathematics itself, in spite of other possibilities of mathematical presentations. And being “mathematics” a word of Greek origin, “matemathike” where “mathema” means understanding, explanation, science, knowledge, learning and “thike” means art or technique according to the etymological dictionary (<http://www.dicionarioetimologico.com.br/matematica/>), some have argued that mathematics is essentially a Greek creation. This is a confusion caused by the etymology of the term later adopted and a creative process whose beginning matches the human presence in the world. However, as noted Archimedes, deductive mathematics is a form of presentation and therefore presupposes a process of creating concepts. Procedural presentation of mathematics reflect this process of creation, approximating the mathematics of the problem in question. Because of this approximation to life, the procedural presentation of mathematics was gradually subdued as a primitive practice, an undeveloped, valueless mathematics. This complex panorama has attained a certain power configuration that was instituted by the Ancient Greeks who overvalued the intellect. As Plato explains in *The Republic*, the highest rank in the government hierarchy of cities should be given to philosophers due to their ability to manage the intellect:

I said: ‘Until philosophers are kings, or the kings and princes of this world have the spirit and power of philosophy, and political greatness and wisdom meet in one, and those commoner natures who pursue either to the exclusion of the other are compelled to stand aside, cities will never have rest from their evils, –nor the human race, as I believe,–and then only will this our State have a possibility of life and behold the light of day.’ Such was the thought, my dear Glaucon, which I would fain have uttered if it had not seemed too extravagant; for to be convinced that in no other State can there be happiness private or public is indeed a hard thing. (Plato 2002, p. 333)

As we can see from the construction of deductive mathematics and its placement as a noble form of mathematics, the process of knowledge construction invariably answers to a particular situation. This applies not only to mathematics but to any field of knowledge and determines two sciences, one legitimized (a State Science) and other marginal (a nomadic science). The State Science is always accompanied by the nomadic science, as a deprecated form of science. Moreover, none of them holds alone (Deleuze and Guatarri 2012, p. 27–28). Archimedes perceived this in the field of mathematics when he asked Eratosthenes to demonstrate his results. Effective possibility of change in this panorama towards the valorisation of life mathematics will only be possible from the second half of the twentieth century, when computers bring to light again the algorithmic way, the “how to”.

Possibilities of Local Mathematics

Although mathematical shapes are historically produced and learned, we see that, traditionally, the presentation of mathematics has been made in a very abstract way, through consolidated formulas, stabilized methods, closed results, omitting its links with life. Even the historical studies of mathematics usually provide an arrangement of results in the timeline, as if mathematical knowledge were built in an everincreasing way. In fact, this is the general conception of mathematical knowledge:

Now we can see what makes mathematics unique. Only in mathematics is there no significant correction—only extension. Once the Greeks had developed the deductive method, they were correct in what they did, correct for all time. Euclid was incomplete and his work has been extended enormously, but it has not had to be corrected. His theorems are, every one of them, valid to this day. Ptolemy may have developed an erroneous picture of the planetary system, but the system of trigonometry he worked out to help him with his calculations remains correct forever. Each great mathematician adds to what came previously, but nothing needs to be uprooted. Consequently, when we read a book like *A History of Mathematics*, we get the picture of a mounting structure, ever taller and broader and more beautiful and magnificent and with a foundation, moreover, that is as untainted and as functional now as it was when Thales worked out the first geometrical theorems nearly 26 centuries ago. (Asimov 1991)

The conception of a ‘‘perfect mathematics’’ is the result of a linear and purified approach in which the interaction with the world is not apparent. This understanding places mathematics as a privileged form of knowledge. But people who are not part of the world that produces this hegemonic knowledge are then placed at a disadvantageous position with relation to this knowledge because they are denied the possibility of questioning the construction of these results. The linear approach, supposedly neutral and universal, bolsters an airtight mathematics, a language understood only in the context of a community of mathematicians, authoritarian, difficult and elitist. It is authoritative because it is imposed to those outside of that collective of mathematicians and who therefore cannot follow its course of construction. Being authoritative, it is also difficult, since questions are only allowed in its own terms (through its own rational justifications), and therefore restricts the space of argumentation to those who have full mastery of this language. And it is also elitist, as it gives the impression that is made only for those alleged to have a special ability.

In the 1980s, European and American anthropologists and sociologists put in check the picture of science presented in this linear manner, that always grows, detached from the world. They entered the laboratories mobilizing the techniques of construction of knowledge that were developed to study the culture of communities called ‘‘primitive’’, proposing to understand how scientific findings are made in the everyday life of laboratories. The proposal was to ‘‘share the collective intimacy’’ of the laboratory, observing practices, making records, going into detail, and thus following the construction of knowledge in its dynamics. It is another epistemological approach: no more knowledge taken as ready, finished, but knowledge in construction, always subject to change. Concerning the production of mathematical knowledge, the sociologist David Bloor put to us the following question:

Everyone accepts that it is possible to have a relatively modest sociology of mathematics studying professional recruitment, career patterns and similar topics. This might just be called the sociology of mathematicians rather than of mathematics. A more controversial question is whether sociology can touch the very heart of mathematical knowledge. Can it explain the logical necessity of a step in reasoning or why a proof is in fact a proof? (Bloor 1991, p. 84)

Such an approach helps to bring out the asymmetric power relations which are manifested by and strengthened in mathematics. These asymmetries are performative, that is to say, they determine mathematical configurations which are often presented and justified by mathematicians as purely technical options. Understanding the historical course of the mathematical shapes and its straight connection with social life and policies enables us to adopt a different position with respect to this knowledge because this allows us to ‘‘share the intimacy’’ of these entities: to question and adapt according to local demands, performing a mathematics that meets the demands of our time and place. In the terms of the mentioned Brazilian Anthropophagic movement of the twenties, this could be called an anthropophagic mathematics.

Brazilian mathematics has grown a lot in this direction since the 1960s and 1970s, when Paulo Freire’s ideas aroused similar proposals in other areas. In the arts, Augusto Boal proposed the Theatre of the Oppressed seeking the social and political change by means of stimulating the critical participation of the audience. Entering in improvised scenes as ‘‘spect-actors’’, the audience would reflect on the oppressive situation in which they were living and propose ways of reversing it (Boal 1985). In mathematics, Ubiratan D’Ambrosio revolted against the imposition of a single, hegemonic mathematics and proposed a programme that sought to recognize the mathematical expressions of the collectives: check how the collective construct explanations for their reality and how they deal with their everyday issues. This was called the ethnomathematics programme. From these ideas, ethnomathematics addresses a set of dimensions, including the epistemologic and educational dimensions, which considers the comprehension of mathematics and local strategies for teaching mathematics in each collective. To avoid taking a stabilized configuration, indifferent to the social, cultural and political changes, ethnomathematics is not supposed to be a new epistemology or general theory of mathematics. It takes the form of situated studies in localities, considering life and people of a specific place.

Over the years, there have been various translations of ethnomathematics, some of them strongly deviating from the initial proposal. Extremely attached to a concept of evolution which takes as a

reference the dominant culture, some translations ended up by taking a philanthropic emphasis, something already surpassed in the field of anthropology and sociology. The ethnographer Franz Boas had already insisted in 1920: “We refrain from the attempt to solve the fundamental problem of the general development of civilization until we have been able to unravel the processes that are going on under our eyes” (Boas 1920).

However, in mathematics, the reference of the dominant culture acquires force because of the tradition of dealing with naturalized, unquestioned and stable mathematical concepts, such as the concept of number. This is precisely the point indicated by the already mentioned laboratory studies regarding mathematics. It requires a certain intimacy with mathematical entities to enlarge the possibilities of alternative constructions. More than this, daring to propose changes to adapt the hegemonic mathematics to fit local needs is only possible from a demystified understanding of mathematics. This was also a point widely studied by Paulo Freire, when he insisted on the need for a problem-based education, not imposed, and away from a cultural invasion (Freire 1974). Precisely, to avoid the imposition of the dominant culture it is necessary to adopt a strategy that gives visibility to the local problems of that collective and uses the proposals of that collective to solve their own problems. Hence, emerges local mathematics and it thus becomes meaningless to match it with hegemonic mathematics, which was designed to solve problems of elsewhere.

Among the various translations that emerged from the proposal of ethnomathematics, Ubiratan D’Ambrosio calls ethnomathematics of everyday life, a type of research that addresses the mathematics from the things of life. This paves the way for situated studies of mathematics, once they leave apparent the links between mathematics and the “world of life”.

A known example of this approach in Brazil is the study of Carraher et al. (1982) named *In Life, Ten: In School, Zero: The Cultural Contexts of Learning Mathematics*. They considered the case of a girl who worked in the sale of coconut in Northeast Brazil, State of Pernambuco. What called attention to these researchers was her ability to calculate the payment of sales without errors. But she was not able to follow the mathematics at school. She knew the algorithms to do the calculations, but she could not apply them because she did not understand how it could give the right result. According to school criteria, she was decidedly a failure in mathematics. Observing the mathematics deployed by the girl, the study concludes what should be obvious that school failure is the failure of the school, and not of the student. The reason for this failure, among other reasons, is given in “the inability to establish a bridge between the formal knowledge that is desired to be transmitted and the practical knowledge which the child, at least in part, already has”, that is, it is lacking the highlighting of the links between ideas and the things of the learner’s life.

For the next example, we start with a special thanks to the teacher Patricia Barbosa, from the school OgaMita, who patiently made it possible for parents of students to understand the way their child was learning the division algorithm. At the same time, she made it possible for parents to understand a procedure that had no meaning for them, except in a mechanical way.

In the city of Rio de Janeiro, a private school for the upper-middle class in the neighbourhood of Tijuca recognizes the need to consider the links between the formal knowledge and the everyday and tries to insert this in school practice. In teaching mathematics, the school OgaMita goes beyond the traditional approaches, such as stimulating the intuition awakened in each example using estimates (what Brazilians call “kiks”, alluding to football kicks towards goal) to give support to the mental calculation and make textual records to undoing the degree of abstraction of algorithms.

To teach the difficult division algorithms to children of about 8 years old, the school adopts the strategy of representing by texts in the algorithm the actions taken at each step during the calculation. Children write sentences about the numbers in the positions which would be occupied only by the numbers in the usual process. Thus, the abstract algorithm becomes self-explanatory because it makes apparent the process of reasoning along the whole calculation. For example, in the process of dividing 120 by 11, the child writes in the place of dividend: “I have 120 balls” and in the place of the divisor: “I want to share among 11 children”. From this, the teacher encourages an estimation process and asks “How many balls do you have for each child?”. Then, the child writes on the site where would be the number of the quotient: “I have given 1 ball to each child”, and where would be the rest of the division, the child

records: “Remained 109 balls”. This process is repeated until reaching the result. At the end of the process, the algorithm presents the complete calculation, with the record of every performed action. As it turns out, it is the same as in traditional mathematics, with only two small changes in the process: (1) the child does not need to hit the quotient on the first try, and it can do many experiments. (2) The child does not lose the links with the performed actions, as is usual when registering only the numbers. After some practice, the written texts become unnecessary and can be deleted from the algorithms. At a later stage, the writings will not be necessary. There will remain only the numbers, as in the traditional procedure. In this learning process, the school mobilizes not only the child but also the families, which are called to participate in this mathematical construction process, reliving processes that for them have always been mechanized.

As a last example of the recognition of local mathematics, we mention a research report that addressed the mathematical knowledge among workers of civil construction. This is a class of extremely low-paid workers and very precarious level of education. Nevertheless, they perform the necessary calculations to erect buildings. The following is the story told by the researcher seeking to understand the mathematics of these workers:

Mister Luis, who attended the literacy class, said to have great difficulty to use algorithms to do the calculations that are necessary in the construction site. He said: “It is difficult, even in the service. Even to make an account would have to be different [...]”. I became interested in what he meant by “would have to be different” and after several explanations I understood that, for example, to determine the half length of a wall, Luis proceeded as follows: He chose a wooden batten visibly greater than half the length of the wall. Next, he positioned this batten at one end of the wall and made a mark with chalk at the end thereof. He proceeded analogously with the other end. At the end of the process, there was a range between the two brands of chalk. Then, with the tape to measure, he determined the middle of this range, which corresponds to the midpoint of the length of the wall that he sought to find. The advantage of such a method, he said, was that the numbers of the determined range “are small and could be calculated by head.” Mister Luis created alternatives to overcome the difficulties he had with “large numbers” and algorithms. He managed thus to overcome the lack of school knowledge (Duarte 2003).

Conclusions

We conclude: reaching the proposal of a mathematical comprehension built on social experience requires a change of attitude regarding the mathematical concepts that have traditionally been taught to us, where mathematics occupies the position of a privileged knowledge whose form and existence is not called into question. It is necessary to perceive that the entities and concepts of mathematics—including all the hegemonic mathematics—are social constructs. To do this we have to become aware of the historical process of building entities and concepts to make it clear that they were developed at a particular time and moment as a response to local demand. Only thus, devoid of this air of universal knowledge, can mathematics take on a plural character allowing compliance to local demands. This approach puts into question the concept of exact sciences: as it is built by humans, from their momentary demands and under the lens of their cultures and their time, the “accuracy” of these sciences is given the same extent that is also given the accuracy of other sciences (human, natural). That is, the accuracy of exact sciences makes sense in relation to a given time and culture. Thus, from its historical process, mathematics becomes contextualized and achievable and opens the way to a plurality of mathematical constructions that may be placed side by side with hegemonic mathematics.

For this special issue on Pluralism in Mathematics, we share with our colleagues of India a few words about mathematics and policies, inspired by Paulo Freire, his experience and ideas. Recognizing the social experience of mathematics enables us to house the mathematical expressions that present the demands of everyday life either in Brazil or in India, the mathematical expression of our people.

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