# Dynamic scheduling of e-sports tournaments

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**Abstract**

Electronic sports tournaments are well adapted to dynamic scheduling. A dynamic approach for scheduling e-sports tournaments based on a modification of the Swiss system design is proposed. We use Colley's method to update all competitors' ratings at every round. The ratings are used for maximizing game fairness and viewers' utility in the integer programming formulation of the team pairing and game scheduling problem solved at each round. The approach was validated using real-life data from the 2020 Honor of Kings World Champion Cup group stage and further evaluated using randomly generated test problems with up to 80 competitors, illustrating the approach's applicability.

*Keywords:* Tournament scheduling, Dynamic scheduling, E-sports, Integer programming, Swiss system, Fairness, Attractiveness, Honor of Kings

## 1. Introduction

### *1.1. Background*

Sports have become a big business in the global economy. Tournaments of different sports are followed on TV channels and the internet by millions of people across the world. Teams make investments in new players. Broadcast rights amount to hundreds of millions of dollars in some competitions. Countries and cities fight for the right to organize worldwide events, such as the Olympics or the FIFA World Cup.

Professional traditional sports involve millions of fans and significant investments in players, broadcast rights, merchandising, and advertising, facing challenging logistics and optimization problems. On the other side, amateur leagues involve fewer investments but require coordination and logistical efforts due to many tournaments and competitors. Electronic sports (e-sports) bring new challenges due to the different nature and designs of their tournaments.

As internet technologies and devices improve by leaps and bounds, electronic sports have seen fast growth in market value and the number of participants in recent years and have a bright future ahead. The General Administration of Sport of China officially launched E-sports as the 99th formal sport in 2003 (Dongsheng et al., 2011). They debuted as an exhibition sport at the Asian Games (Wade, 2018) in 2018, meaning that the Olympic Committee has recognized them

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as a formal sports program (Hallmann & Giel, 2018). The Olympic Council of Asia announced on April 17, 2017, that e-sports would become one of the 37 official sports in the 2022 Hangzhou Asian Games (delayed to 2023): there would be eight medal events, which are PUBG Mobile, DOTA 2, Hearthstone, League of Legends, FIFA, Street Fighter V, Arena of Valor, and Dream of the Three Kingdoms 2.

According to a report by Insider Intelligence (Intelligence, 2022), a subscription-based market research company, the monthly viewers for e-sports would reach 29.6 million in 2022, an increase of more than 11.5% compared to 2021. It was also estimated that e-sports enthusiasts would grow by a compound annual growth rate of 8.1% from 2020 to 2025 and surpass 640 million in 2025 (Newzoo, 2022). The global revenues generated by e-sports in 2021 have reached more than one billion US dollars (Statista, 2022), of which the Chinese market accounted for nearly 20%. Moreover, the total prize pools of some popular e-sports tournaments have surpassed that of traditional sports. For example, the prize pool for the 2021 International 10 (TI10) reached 40 million US dollars, while for the 2022 NBA playoffs, it amounted to 17.4 million US dollars; see Table 1 for more details. The approximate value of the prize pool for the UEFA Champions League was obtained from (Chase Your Sport, 2021), see (Esports Earnings, 2022; Heath, 2022) for the highest prize pools for e-sports.

Good schedules play a vital role in a sports tournament. Furthermore, they can significantly improve their fairness and logistics while reducing the organizers' costs. Although some organizers of electronic sports tournaments use scheduling methods initially developed for traditional sports, there are significant differences between the scheduling of tournaments of traditional sports and e-sports. Traditional scheduling methods are only sometimes appropriate for e-sports due to their competitions' nature and the tournaments' formats. First, tournaments of most traditional sports follow the round-robin design, while e-sports tournaments may follow the Swiss-system design. Second, because the composition of e-sports teams changes frequently, static scheduling policies may be unfair due to not considering such changes. Third, few existing tournament scheduling methods search for optimal pairings of the teams that will meet in each round of the competition. Teams participating in e-sports tournaments may play more than one game in a single day, while in traditional sports, teams usually play at most once a day. In summary, different tournament formats and conditions require new developments and methods for scheduling e-sports tournaments (Pizzo et al., 2022).

In this article, we propose, develop, and evaluate dynamic scheduling methods for e-sports tournaments that use optimization methods to select the opponents that will meet in each round and to determine the order (i.e., the time slots) in which the games will be played. We consider a Swiss-system tournament with an even number of $n$ participating teams organized in a fixed number $R$ of rounds. Every team plays exactly one game in each round. The games in each round are played in $n/2$ sequential, non-overlapping time-slots. The best-ranked team after the last round is the winner.

### *1.2. Literature review*

The main problem in sports scheduling consists in determining the date (or round), time, and venue (facility, stadium, court, arena) in which each game of a tournament will be played. Applications exist in scheduling real-life sports tournaments such as football, baseball, basketball, cricket, and hockey. Different exact and approximate approaches, including integer programming, constraint programming, metaheuristics, and hybrid methods, have solved these problems.

Table 1: Prize pools for some e-sports and traditional sports tournaments.

| E-sports | Prize pool (USD million) | Traditional sports | Prize pool (USD million) |
|---|---|---|---|
| 2021 International 10 (Dota 2) | 40.0 | 2020 UEFA Champions League (soccer) | 2379.0 |
| 2019 Fortnite World Cup Finals | 30.4 | 2021 Formula 1 (car racing) | 797.0 |
| 2021 Honor of Kings World Champion Cup | 7.7 | 2021 MLB World Series (baseball) | 90.5 |
| 2021 PUBG Global Invitational.S | 7.1 | 2022 IPL (cricket) | 59.6 |
| 2018 League of Legends World Championship | 6.4 | 2021 US Open (golf) | 57.5 |
| 2020 Call of Duty League Championship | 4.6 | 2021 Australian Open (tennis) | 49.2 |
| 2021 PUBG Global Championship | 4.4 | 2020 NHL Stanley Cup playoffs (hockey) | 32.0 |
| 2018 Fortnite Fall Skirmish Series | 4.0 | 2022 NBA playoffs (basketball) | 17.4 |
| 2019 Overwatch League playoffs | 3.5 | 2022 Super Bowl (football) | 11.9 |
| 2015 Dota 2 Asia Championship | 3.0 | 2021 Tour de France (cycling) | 2.3 |

There are many relevant aspects to be considered in the determination of the best schedule for a tournament. In some situations, one seeks a schedule minimizing the total traveled distance, as in the case of the traveling tournament problem (Easton et al., 2001) and in that of its mirrored variant (Ribeiro & Urrutia, 2007a), which is common to many soccer tournaments in SouthAmerica (Durán et al., 2007). Other problems attempt to minimize the total number of breaks, i.e., the number of pairs of consecutive home games and away games played by the same team. Minimizing the value of the carry-over effects (Russell, 1980; Melo et al., 2009) is another fairness criterion leading to an even distribution of the sequence of games along the schedule. Some problems in sports scheduling have a multicriteria nature. Ribeiro & Urrutia (2007b, 2009) tackled the scheduling of the annual Brazilian soccer tournament formulated as a bicriteria optimization problem, where one of the objectives consisted in maximizing the number of games that could be broadcast by open TV channels (to increase the revenues from broadcast rights). The other consisted of finding a balanced schedule with a minimum number of home breaks and away breaks (for fairness).

Ribeiro (2012) provides an introductory tutorial to the main problems in sports scheduling, also covering the main practical applications. Although being more focused on the problems and applications, it also addresses the most often used solution methods and algorithmic approaches applied in their solution. It should be considered a starting point for newcomers and research in this area. The interested reader is referred to Rasmussen & Trick (2008) for a comprehensive survey of the literature on round-robin tournament scheduling and to Kendall et al. (2010) for an annotated bibliography of scheduling problems in sports. A framework for constrained sports scheduling problems was introduced by Nurmi et al. (2010).

Parshakov (2019) studied the elements that affect the performance of different teams in a tournament and the utility of the viewers. The main elements include fairness (Bao et al., 2017), the suspense utility (Chan et al., 2009), and the willingness to win for each team (Palomino & Rigotti, 2000). The difference between the rewards received for winning or losing a game may also affect the competitors' performance. Cheng et al. (2019), Coates & Parshakov (2016), and Moldovanu & Sela (2001) studied the optimal prize allocation considering various effort cost functions. In practice, fans enjoy and are attracted by teams and players that make a more strenuous effort to win. In our work, we do not incorporate prize allocation into our analysis since it does not affect the dynamic scheduling model. A more sophisticated distribution of the total prize pool between the teams might improve the efforts made by the teams, which we leave for future work.

Rules used in sports tournaments can be generalized and applied to other problems involving the selection of one or more winners out of many participants. For example, the auction and the crowdsourcing problems can use similar rules for the winner's decision. Using the differences in the rewards to motivate the competitors has been studied and quantified in the context of sports competitions by Palomino & Rigotti (2000) and in elimination contests for crowdsourcing by Hou & Zhang (2021), a significant problem for sponsors in the digital era (Al Mashalah et al., 2022).

Chikish et al. (2019) supported previous evidence in the literature emphasizing the complementarity between e-sports and traditional sports. However, many differences exist between them (Lee & Schoenstedt, 2011) and e-sports differ from traditional sports in several dimensions. First, there are differences in the athletes' physicality: e-sports involve movements using small groups of muscles, whereas traditional sports require large groups of muscles. Jenny et al. (2017) noticed that e-sports do not require the same level of physicality associated with standard definitions of sports. Second, sport is conceived as an area organized and regulated by institutions (such as FIFA in the case of soccer). Although one of the five criteria defined by the Global Association of International Sports Federations states that "sport should not rely on equipment that a single supplier provides," e-sports rely on commercial products owned and governed by private organizations (Karhulathi, 2017). Third, traditional sports and e-sports competitions are usually played using distinct tournament formats in different environments: while traditional sports competitions are usually played in open fields, courts, or large arenas, most major e-sports tournaments are held in venues that can hold tens of thousands of spectators and are streamed online.

Effective rating methods are necessary for dynamic scheduling since we seek to pair teams with similar performance in the competition. Different methods exist for rating the teams in a tournament based on their strengths, such as multi-criteria decision-making (Pradhan & Abdourazakou, 2020) and Colley's ranking method (Colley, 2002). Bouzarth et al. (2020) proposed to produce NFL schedules that combine some traditional elements with dynamically scheduled games, optimizing different objectives. Among them are reducing the variability of the teams' strength of schedule or minimizing the pairwise comparisons needed to differentiate team quality to make each team's regular season schedule as fair as possible. These concerns are similar to those of the dynamic scheduling approach for e-sports tournaments proposed here.

### *1.3. Main contributions and text organization*

This work describes the first application of optimization methods to the scheduling of e-sports tournaments, which are particularly suitable for dynamic strategies, as illustrated in Section 1.1. Its main methodological contribution is a new dynamic scheduling strategy for e-sports tournaments based on the Swiss system design.

The Swiss system design allows for a more significant number of teams in the competition. It has a shorter duration than the round-robin design applied in most professional leagues of traditional sports such as soccer. Compared with knockout tournaments, it also avoids the premature elimination of competitors with a few bad results.

The dynamic scheduling approach was validated with real-life data collected from the 2020 Honor of Kings World Champion Cup and randomly generated large-size test problems with up to 80 competitors. The integer programming problem in each round can be solved fast. The numerical results are stable and show that the tournament's winner can be obtained in $\lceil \log_2 n \rceil$ or a few more rounds, where $n$ is the number of competitors.

This article is organized as follows. Section 2 describes the two basic tournament designs (round robin and knockout), with their advantages and drawbacks, as well as the Swiss system design that is very suitable for dynamic scheduling in the context of electronic sports. Section 3 describes and summarizes Colley's method to update the ratings of all teams round by round in a context where the competitors have schedules of different strengths. Section 4 describes the integer programming formulation of the dynamic scheduling approach used at each round for team pairing and game scheduling. The ratings produced by Colley's method are used for maximizing game hardness and viewers' utility in the integer programming problem solved at each round. Section 5 reports computational experiments and numerical results for real-life data collected from the 2020 Honor of Kings World Champion Cup (KCC2020) and for randomly generated problems of much larger size, illustrating the approach's applicability. Concluding remarks are drawn in the last section.

## 2. Tournament designs

Different designs exist for a sporting contest or tournament. A tournament's design (or format) may influence its outcome uncertainty, the number of unimportant matches within the tournament, and even the fairness of the tournament (Scarf et al., 2009). Owen & Weatherston (2004) suggest that low uncertainty of outcome may mean that fan, broadcaster, and sponsor interests will not be maximized. This section summarizes the two basic tournament designs (round robin and knockout), from which all other designs are derived or hybridized, and discusses their main pros and cons. It also discusses Swiss system tournaments, the only tournament design variant appropriate for dynamic scheduling and particularly useful in the context of electronic sports.

### *2.1. Round robin tournaments*

Each competitor in a *round-robin tournament* (or *all-play-all*) plays the same number of games with every other competitor. Most professional leagues of traditional sports play either a single round-robin tournament (each pair of competitors meets exactly once) or a double round-robin tournament (each pair of competitors meets twice). Generally, each competitor plays against every other exactly $m$ times. The number of rounds in a *compact* tournament (where each competitor plays precisely once in each round) is equal to $m(n-1)$, where $n$ is an even number of competitors.

In principle, a round-robin tournament would be the fairest way to determine the champion or winner from among a known and fixed number of competitors. Each of them has equal chances against all other competitors. There is no initial seeding of competitors precluding a game between any given pair. Every competitor plays against every other, and competitors are not eliminated after a certain number of losses.

However, round-robin tournaments have many more games compared to other tournament designs (see Section 2.2) and, in consequence, can be very long, limiting the number of competitors in such tournaments. Professional leagues whose competitions are played as round-robin tournaments are usually limited to a few more than 20 competitors. Furthermore, they can suffer from later scheduled games potentially not having any substantial importance, which also opens the door to easier manipulations of results. Also, many games may be uninteresting due to the imbalance between the opponents, and no effective strategy exists to maximize game-by-game tournament attractiveness.

Competitors accumulate points along the tournament depending on their results (wins, draws, or losses) and, in some cases, their game scores. However, ranking the competitors simply by using their accumulated points or scores may lead to distortions. Various approaches have been studied for ranking the competitors of round-robin sports tournaments. Jech (1983) and Keener (1993) handled the ranking problem by calculating the eigenvector of a linear system. They provided ranking-existence and uniqueness conditions based on the comparability of each pair of competitors, which is captured by the Perron-Frobenius theorem (Perron, 1907; Frobenius, 1912). Thompson (1975), Leake (1976), and Knorr-Held (2000) used traditional paired comparison approaches that rely only on the win/loss record of each competitor. They define the winning probability of the competitors in each game based on their ability and ranking difference. Then, they seek the optimal ranking and parameters based on maximum likelihood methods, which make the tournament's results to be those with the highest probability. Goddard (1983) represented the tournament results by a graph and developed a method for ranking the competitors based on paths and circuits in this graph. A strength vector is defined for the teams and updated iteratively, incorporating the opponents' strengths. Ali et al. (1986) advanced Goddard's work on minimum violations ranking, while Stob (1985) challenged Goddard's method and championed Thompson's.

Some e-sports tournaments are played following the round-robin design. One of the most well-known e-sports round-robin tournaments is the annual League of Legends World Championship. Participants compete for the crown of the World's best League of Legends team. The winner earns a multi-million dollar grand prize. A double round-robin tournament determines the best two teams, followed by a final game between them. Other e-sports tournaments that have been played using the round-robin format include The International (Dota 2), the Honor of Kings World

Champion Cup, the ESL Pro League, the regular seasons of Overwatch League, and the Rainbow Six Siege.

### *2.2. Knockout tournaments*

A *knockout tournament* for $n$ (generally, a power of two) competitors is a *single elimination tournament* where the loser of each game is immediately eliminated. Each winner will play with another winner in the next round until the final match-up, whose winner becomes the champion. Some variants include double- or triple-elimination, or a combination with a best-of-$k$ series, when a competitor is eliminated after losing $(k+1)/2$ out of $k$ games with the same opponent, where $k$ is odd.

The single-elimination design allows many more competitors to participate in the tournament than in the round-robin format. Every game counts and is equally relevant for the two competitors: the winner gets all, and the loser is eliminated. However, most competitors are eliminated after relatively few games. One-half of the competitors still in contention is eliminated at the end of each round. Variants such as the double-elimination format are tentatively used to overcome this disadvantage.

Seeding and bracketing are very relevant to the outcome of the tournament. For a given seeding and bracketing strategy, the games to be played in the first round are directly set. The results of the current round automatically determine the games to be played in the next one. Seeding and bracketing define the schedule.

Knockout tournaments have been studied, e.g., by Hwang (1982), Horen & Riezman (1985), and Hennessy & Glickman (2016), among others. For e-sports, best-of-$k$ series are generally used together with the knockout design during the playoffs. Since there may exist many uncertainties about the players' performance, knockout tournaments are rarely used in the group stage of e-sports.

Grand Slam tennis tournaments use the single elimination design. The players are seeded in different brackets according to their rankings. If the best-ranked opponent wins every game, the final match-up will see the two best-ranked players face to face. Other competitions may have two phases, with the second being a knockout stage, often called playoffs. One of the most noticeable sports tournaments organized with this design is the soccer FIFA World Cup.

### *2.3. Swiss system tournaments*

A *Swiss-system tournament* between an even number $n$ of teams is a non-eliminating tournament design featuring a fixed number of rounds of competition, which is considerably shorter than a round-robin tournament. Each competitor does not play against every other. Competitors are paired in each round using a set of rules designed to ensure that each competitor plays an opponent with a similar score (or rating), while avoiding as much as possible playing the same opponent more than once (i.e., avoiding game repetitions) over the tournament. After all rounds, the winner is the best-ranked competitor, i.e., that with the best aggregate score (or rating). This tournament design was featured for the first time in 1895 at a chess tournament in Zurich, Switzerland, which is how it earned its name.

The Swiss system is valuable and appropriate for tournaments where (1) there are too many competitors for a whole round-robin tournament to be feasible in practice due to timing constraints, and (2) eliminating any competitors before the end of the tournament is undesirable.

Chess tournaments are often organized following the Swiss system. The competitors go for a predetermined number of rounds, with two competitors competing head-to-head in each round. No competitor is eliminated during the tournament, but some competitors may never face each other. Two emerging issues in Swiss system tournaments are (1) how to pair the competitors and (2) how to rank them, considering their previous results. The pairing algorithm is applied in each round and aims to pair competitors with similar performance, measured by their wins and draws (see, e.g., (International Chess Federation, 2020) for details).

E-sports tournament organizers have been using a variant of the Swiss system over the years. The participants play until they reach a predetermined number of wins or losses instead of having them play the same number of games. In this variant, a competitor that wins the required number of games advances to the next tournament stage, while one that loses a predetermined number of games is eliminated. This format has been implemented in Dota 2, Rocket League, Hearthstone, and, most notably, in Counter-Strike: Global Offensive (CS:GO) tournaments, where the norm has been to require three wins to advance and three losses to be eliminated.

### *2.4. Swiss system vs. round robin designs for e-sports tournaments*

The round-robin format is unsuitable for e-sports tournaments with many participating competitors. The Swiss-system design, or some of its variants, in which relatively few rounds will be played, is likely to be the most appropriate tournament design and is explored in this work.

**Remark 1.** To further illustrate the claim in the above paragraph, we assume a tournament played by an even number $n$ of teams. Both round-robin and Swiss system tournaments are usually organized in synchronous rounds of $n/2$ games, where every competitor plays precisely once. Since in a (single) round-robin tournament, each competitor plays every other team exactly once, $n-1$ rounds are necessary to determine its winner. Empirical evidence and theoretical results about winner determination by pairwise comparisons show that the number of rounds necessary to determine the winner of a Swiss-system tournament requires about $\lceil \log_2 n \rceil$ rounds, which is much smaller than the $n-1$ rounds of a single round-robin tournament. □

Unlike round-robin or other designs where all pairings are known beforehand, in a Swiss-system tournament, the scheduling is dynamic, and the results of the current round determine team pairings for the next one. Csatò (2013, 2017) present detailed descriptions of the Swiss-system and its rating (based on the number of wins) and pairing (games formed by competitors with the same number of wins, no repetitions) strategies.

Ranking the competitors in Swiss-system tournaments involves two main challenges. First, the possible existence of circular triads, when player A has won B, player B has won C, but player C has won A. Second, incomplete comparisons because the number of rounds is smaller than $n-1$, and the competitors have schedules of different strengths since each competitor does not play against every other. Consequently, the final ranking may not result in all competitors forming a linear order (a complete, transitive, and antisymmetric binary relation).

The Swiss system imposes a dynamic scheduling strategy (i.e., team pairing in each round), where the games of a new round become known after all games of the current one terminate. This characteristic imposes logistic difficulties in the case of extramural tournaments where each team has its stadium because decisions associated with traveling and displacement of teams (as well as those related to security, media coverage, and displacement of the fans) will have to be made quickly on-the-fly. However, the Swiss system is appropriate for large tournaments held in a single place (such as a large arena complex with one or more game-playing facilities) or over the internet (such as electronic sports). This aspect is desirable because dynamic game scheduling opens new horizons for creating more attractive and competitive tournaments for e-sports.

**Remark 2.** A critical issue to be decided in Swiss-system tournaments is the number of rounds that should be played, assuming that all competitors play exactly one game in each round. We

Table 2: Main variables, properties, and references for different tournament designs.

| Tournament rules | Representative references | Scheduling variables | Properties |
|---|---|---|---|
| Round robin (single/double) | Harary & Moser (1966) Rasmussen & Trick (2008) | Round and venue of each game | No teams eliminated. Any team plays every other the same number of times. |
| Knockout (single/double elimination) | Hwang (1982) Horen & Riezman (1985) Hennessy & Glickman (2016) | Opponents in each game (team pairing) | Teams eliminated after a certain number of losses. No optimization of team pairing. |
| Swiss system (traditional) | Csatò (2013, 2017) | Opponents in each game (team pairing) | No teams eliminated. No optimization of team pairing. |
| Dynamic scheduling based on the Swiss system design | (this work) | Games played in each round (team pairing) and time slot of each game (scheduling) | No teams eliminated. Attractiveness optimization for game pairing. Fairness constraints for time slot determination. |

also assume that $1 \leq k \leq n$ is the desired number of top competitors that should be ranked as the tournament outcome. Furthermore, in each game, the winner gets one point and the loser none (there are no draws). Therefore, we seek an algorithm that ranks in the least number of required rounds $R(n,k)$ the top $k$ competitors out of $n$. Iványi & Fogarasi (2017) showed that if the transitivity rule holds and the number of competitors is even and greater than four, then a pairing algorithm exists that determines the complete ordering of all competitors in exactly two rounds. However, the transitivity rule is hardly observed in practice for any sport. Since the largest among $n$ integer numbers can be obtained with $O(\log n)$ comparisons (see, e.g., (Baddar & Batcher, 2012)), at least $\lceil \log_2 n \rceil$ rounds will be necessary to find the champion of a compact sports tournament in which each competitor plays in all rounds. The round-robin design provides an upper bound for determining the full ordering, i.e., $R(n,1) \leq R(n,2) \leq \ldots \leq R(n,n) = n-1$. We have seen that the knockout format provides the strongest competitor very efficiently with a minimum number of comparisons, i.e., $R(n,1) = \lceil \log_2 n \rceil$ (see also (Knuth, 1998)). Since the second strongest competitor must have been eliminated by the winner directly in one of the $\lceil \log_2 n \rceil$ rounds, we obtain that $R(n,2) = \lceil \log_2 n \rceil + \lceil \log_2 \lceil \log_2 n \rceil \rceil$. Consequently, depending on the tournament's requirements and the time available for its completion, the organizers usually set the number of rounds between $\lceil log_2 n \rceil$ and $n-1$. We illustrate in Section 5 the sensitivity of the proposed dynamic scheduling approach to the number of rounds and we show that setting the number of rounds to $\lceil log_2 n \rceil$ is a very appropriate choice. □

We summarize in Table 2 a comparison of the most typical tournament designs described in Sections 2.1 to 2.3. It includes the main variables and properties of their scheduling models and our newly proposed dynamic scheduling approach, together with relevant references to each. In addition, Table 3 illustrates some design details for the group and final stages of some real-life e-sports tournaments.

## 3. Rating teams with schedules of different strengths

We observed in Sections 2.3 and 2.4 that the Swiss system seeks to create pairings of competitors with a similar score (or rating) in each round.

Table 3: Tournament designs for the group and final stages of some e-sports tournaments.

| E-sports tournaments | Group stage | Final stage |
|---|---|---|
| 2022 The International (Dota 2) | Single round robin | Double elimination |
| 2022 Overwatch League | Single round robin | Double elimination |
| 2022 Rainbow Six NAL | Single round robin | Single elimination |
| 2022 ESL Pro League | Single round robin | Single elimination |
| 2021 Honor of Kings World Champion Cup | Single round robin | Single elimination |
| 2022 League of Legends World Championship | Double round robin | Single elimination |
| 2022 Valorant Champions | Double elimination | Double elimination |
| 2022 Call of Duty League Championship | Points system | Double elimination |
| 2021 PUBG Global Championship | Points system | Points system |
| 2019 Fortnite World Cup Finals | Points system | Points system |
| 2022 IEM Rio Major (CS:GO) | Swiss system | Single elimination |
| 2022 Rocket League Championship Series | Swiss system | Single elimination |

Although simple statistics such as the number of wins would suffice to produce a fair rating if all competitors in a tournament played schedules of similar strength, such as in a round-robin tournament, this is not the case for the Swiss system. First, each competitor plays against a different minor subset of competitors and only against some participating competitors. Second, the competitors play schedules of different strengths. Consequently, establishing a ranking is more complex if consistency is sought.

**Remark 3.** We illustrate how simple statistics may lead to similar ratings for competitors with schedules of entirely different strengths. For example, two competitors, A and B, may have three wins each over very different subsets of competitors. Suppose that competitor A defeated three strong competitors, while competitor B defeated three weak competitors. If only wins were counted, then both would have the same rating (i.e., three wins), although competitor A certainly seems to be stronger than B. □

Colley's rating method (Colley, 2002; Pasteur, 2010; Stewart, 2013) exhibits several beneficial properties to overcome the difficulties of rating the strength of all competitors in a tournament. This approach is simple and keeps track only of wins and losses. This feature avoids the reliance on scores that generate some dependence on margins that may be trickier, mainly when the competitors have schedules with divergent strengths, as in the case of the Swiss system, where each competitor plays only against a small fraction of the participating competitors. It also eliminates (i) any bias toward history or tradition, (ii) the need to invoke some ad hoc means of deflating runaway scores, and (iii) the use of any other ad hoc adjustments, such as home/away tweaks in the case of extramural tournaments.

A detailed description of the Colley matrix method appears in Colley (2002). The rating $r_i$ of each competitor $i$ at some round of the competition is given by

$$r_i = \frac{1 + wins_i}{2 + total_i}, \tag{1}$$

where $wins_i$ and $total_i$ are, respectively, the number of wins and the total number of games played by competitor $i$. Therefore, at the beginning of the season, all competitors have an equal rating of 1/2. After winning one game in the first round, a competitor improves its rating to 2/3, while the rating of a losing competitor becomes 1/3.

The number of wins of competitor $i$ in equation (1) may be rewritten as

$$wins_i = \frac{wins_i - losses_i}{2} + \frac{total_i}{2} = \frac{wins_i - losses_i}{2} + \sum_{1}^{total_i} \frac{1}{2}, \tag{2}$$

where $losses_i$ denotes the number of losses of competitor $i$ and the last term $\sum_1^{total_i} 1/2$ corresponds to the sum of the ratings of $total_i$ random competitors. If these random competitors are replaced by the first $total_i$ opponents of competitor $i$, and we use their actual ratings, then we obtain the effective number of wins of competitor $i$:

$$wins_i^{eff} = \frac{wins_i - losses_i}{2} + \sum_{j=1}^{total_i} r(i,j), \tag{3}$$

where $r(i,j)$ is the rating of the opponent of competitor $i$ in round $j$. The second term (the summation) in equation (3) is the adjustment for the strength of the specific schedule played by competitor $i$. All competitors' ratings are initialized with 1/2 and computed iteratively at each round up to a specific numerical tolerance using equation (3).

A more elegant and efficient solution approach consists in rewriting equations (1) and (3) as follows, by setting $wins_i = wins_i^{eff}$ for every competitor, as in the outcome of the iterative approach:

$$(2 + total_i)r_i - \sum_{j=1}^{total_i} r(i,j) = 1 + \frac{wins_i - losses_i}{2}, \qquad i = 1, \dots, n, \tag{4}$$

which is, for each round, a system of $n$ linear equations with $n$ variables, each associated with one participating competitor. The above equation may be written in matrix form as follows:

$$C \cdot r = b, \tag{5}$$

where $r$ is the $n \times 1$ column vector formed by all ratings $r_i, i = 1, \dots, n$, and $b$ is the $n \times 1$ column vector associated with the right-hand-side of equation (4):

$$b_i = 1 + \frac{wins_i - loss_i}{2}, \qquad i = 1, \dots, n; \tag{6}$$

and $C = \{c_{ij}\}$ is the $n \times n$ Colley matrix with coefficients

$$c_{ii} = 2 + total_i, \qquad i = 1, \dots, n, \tag{7}$$

$$c_{ij} = -games(j,i), \qquad i,j = 1, \dots, n : i \neq j, \tag{8}$$

where $games(j,i)$ is the number of times competitor $i$ has played competitor $j$.

Solving the linear system (5) with the coefficients given by equations (6)–(8) is the method for computing the ratings $r_i$ for every competitor $i = 1, \dots, n$ in each round. Existing solvers can directly solve it by Cholesky decomposition and back-substitution because matrix $C$ is symmetric, real-valued, and positive definite. Existing solvers can directly solve it when the number of competitors is small. The Colley matrix method conserves an average of 1/2. All ratings belong to $(0,1)$ (Colley, 2002).

In addition to other characteristics mentioned above, some advantages of this approach are that (i) it is reproducible, (ii) it uses a minimum of assumptions, (iii) it ignores runaway scores, and, very important, (iv) it adjusts for the strength of the schedule.

We applied Colley's method mainly because it is appropriate when the competitors play schedules of entirely different strengths. However, other rating methods exist and could be used for dynamic scheduling. We mention Elo's method (Elo, 1961) and its variant (Elo, 1978) that make use of a probability-based rating system; Massey's least squares method (Massey, 1997), which assumes that the difference between the ratings of two teams may predict the score of the game between them and involves the solution of a least squares problem; and the Markovian method (Redmond, 2003; Mattingly & Murphy, 2010) that uses graph theory and Markov chains to generate ratings of the objects in a finite set.

**Remark 4.** Colley's rating method assumes that each game has a winner. Other rating methods could be used for pairing the competitors of sports (such as soccer) tournaments whose games may end up in a draw. □

## 4. Dynamic scheduling by integer programming: Team pairing and game scheduling

The attractiveness (or demand or interest) of a game in a league depends on the competitive balance of the tournament (Palomino & Rigotti, 2000), measured by the uncertainty of its outcome: fans enjoy sporting events more if the winner is not easy to predict. In other words, the more similar the competitors' winning chances, the more exciting the tournament is.

The ratings produced by Colley's method described in Section 3 will be used for round-by-round dynamically pairing the teams participating in the tournament following the Swiss-system design. To maximize the tournament's attractiveness, pairings in each round will involve teams with similar ratings as much as possible. In addition, to maximize the fairness of the tournament, repetitions of the same game in consecutive rounds will be avoided except when necessary to ensure feasibility.

To assess the tournament's competitive balance, we define the *unattractiveness* of the game between teams $i$ and $j$ at some round as the squared difference of their ratings. The more similar the ratings of the two teams are, the more attractive (or less unattractive) the game between them is:

$$u_{ij} = (r_i - r_j)^2. \tag{9}$$

The more balanced the game between teams $i$ and $j$, the closer the value of $u_{ij}$ is to zero. Therefore, we seek to minimize the sum of $u_{ij}$ over the $n/2$ games in each round to increase the tournament's competitive balance.

We observe that there is no overlapping between any pair of games played in the same round: the games are played in $n/2$ sequential time slots. All games can be streamed and followed online by the audience, and they can take place at the same arena one after the other.

We consider a formulation that simultaneously optimizes team pairing (i.e., opponent selection) and game scheduling (i.e., the order in which the games are played in each round). We define the following decision variable:

$$x_{ij}^{kt} = \begin{cases} 1, & \text{if team } i \text{ plays with team } j \text{ in time slot } t \text{ of round } k, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

The goal consists in maximizing the interest (which amounts to minimizing the unattractiveness) of the games in each tournament round $k = 2, \ldots, R$, which can be achieved by scheduling attractive games between teams with similar ratings. The number $R$ of rounds is an external parameter set by the tournament organizers and is known beforehand. The pairings are randomly made in the first round because there is no previous information about the team ratings. Thus, the problem to be solved at each round $k = 2, \ldots, R$ is formulated as follows:

$$\underset{x_{ij}^{kt}}{\text{minimize}} \quad \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{t=1}^{n/2} u_{ij}^{k} x_{ij}^{kt} \tag{11}$$

$$\text{subject to:} \quad x_{ij}^{kt} = x_{ji}^{kt}, \qquad i, j = 1, \ldots, n : i \neq j; t = 1, \ldots, n/2, \tag{12}$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{t=1}^{n/2} x_{ij}^{kt} = 1, \qquad j = 1, \ldots, n, \tag{13}$$

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ij}^{kt} = 2, \qquad t = 1, \ldots, n/2, \tag{14}$$

$$\sum_{t=1}^{n/2} x_{ij}^{kt} \leq 1 - \sum_{\ell=1}^{\min\{L,k-1\}} \sum_{t=1}^{n/2} \bar{x}_{ij}^{k-\ell,t}, \qquad i, j = 1, \ldots, n : i \neq j, \tag{15}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{t=1}^{n/2} t(x_{ij}^{kt} - \bar{x}_{ij}^{k-1,t}) \leq D, \qquad i = 1, \ldots, n, \tag{16}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{t=1}^{n/2} t(\bar{x}_{ij}^{k-1,t} - x_{ij}^{kt}) \leq D, \qquad i = 1, \ldots, n, \tag{17}$$

$$x_{ij}^{kt} \in \{0, 1\}, \quad \forall i, j = 1, \ldots, n; i \neq j; t = 1, \ldots, n/2. \tag{18}$$

The objective function (11) minimizes the sum of the squared differences of the ratings of the teams paired in round $k$, i.e., it maximizes the overall attractiveness of the games played in this round. The symmetry constraints (12) establish that the game between teams $i$ and $j$ is the same as the one between teams $j$ and $i$. Constraints (13) ensure that every team plays precisely once in round $k$. The non-overlapping constraints (14) guarantee that exactly one game will be scheduled in each of the $n/2$ time slots of round $k$. Inequalities (15) are the no-repeaters constraints: they ensure that there should be at least $L$ rounds between any two consecutive games between teams $i$ and $j$ ($L = 0$ means that two games between teams $i$ and $j$ may take place in two consecutive rounds). Constraints (16)-(17) enforce that the distance between the positions of the games played by each team in the previous and current rounds is bounded from above by a given parameter $D$ (the maximum time-slot difference, or distance). Constraints (18) express the integrality of the decision variables.

**Remark 5.** We recall that model (11)-(18) above denotes the problem solved at each round $k = 2, \ldots, R$. Note that each $\bar{x}_{ij}^{k-\ell,t}$ in this formulation does not represent a problem variable, but instead the actual value of the variable $x_{ij}^{k-\ell,t}$, already computed for the previous round $k - \ell$. □

This formulation involves two parameters: $L$ and $D$. Parameter $L$ is the minimum number of rounds between two consecutive games between the same pair of teams. $L = 0$ corresponds to an unconstrained problem. $L = R-1$ corresponds to the more challenging case in terms of feasibility, where there is at most one game between any pair of teams (no game repetitions allowed).

Parameter $D$ is the maximum difference (or distance) between the time slots of the games played by any team in rounds $k$ and $k - 1$. Since the only feasible solution for $D = 0$ would correspond to the repetition in round $k$ of the same games played in round $k-1$ in the same order, this case may be disregarded. $D = 1$ corresponds to the case where the time slot of the game played by each team may change by at most one unit between rounds $k$ and $k-1$. $D \geq n/2 - 1$ corresponds to the unrestricted case.

## 5. Numerical experiments

All algorithms were implemented in Visual Studio 2019/C++ with the toolset Visual Studio 2017 (v141) and integrated with CPLEX Studio version 12.9. The computational experiments were performed on an Acer Aspire E15 Touch machine with four Intel cores I5-5200U, 2.20 GHz processors, and 16GB of RAM running under version 10.0.19044.1766 of the Microsoft Windows 10 operating system.

Two sets of computational experiments have been performed. Section 5.1 reports results for real-life data collected from the 2020 Honor of Kings World Champion Cup (KCC2020). Section 5.2 reports results for randomly generated problems of larger size. There are no previous benchmarks available to be used for straightforward comparisons since dynamic scheduling was not applied to this problem before.

### *5.1. Simulated results for dynamic scheduling from realistic data*

In this section, we apply the newly proposed dynamic scheduling approach based on the Swiss system using real-life data collected from the 2020 Honor of Kings World Champion Cup (KCC2020) (Liquipedia, 2021) with 12 teams. KCC2020 was organized into two groups (A and B) of six teams each. Each group played a single round robin. The actual results for Groups A and B games of KCC2020 appear in Tables 4 and 5, respectively.

Table 4: Actual results for the games of the six teams in Group A of KCC2020.

| | QG Happy | AG Super Play | eStar Pro | Team WE | Rox Gaming |
|---|---|---|---|---|---|
| Turnso Gaming | 2x1, July 26 | 2x1, July 15 | 1x2, July 17 | 2x0, July 22 | 2x0, July 18 |
| QG Happy | | 2x0, July 18 | 2x0, July 19 | 0x2, July 15 | 2x0, July 22 |
| AG Super Play | | | 2x0, July 26 | 2x0, July 25 | 2x0, July 23 |
| eStar Pro | | | | 2x0, July 16 | 2x0, July 24 |
| Team WE | | | | | 2x0, July 19 |

Table 5: Actual results for the games of the six teams in Group B of KCC2020.

| | ViCi | Douyu | TTG X-Quest | Ghost Owl | Esports of Macau |
|---|---|---|---|---|---|
| Mighty Tiger | 0x2, July 23 | 2x1, July 15 | 2x0, July 22 | 1x2, July 19 | 0x2, July 16 |
| ViCi | | 1x2, July 17 | 2x0, July 16 | 2x1, July 24 | 1x2, July 25 |
| Douyu | | | 2x0, July 18 | 1x2, July 25 | 2x0, July 24 |
| TTG X-Quest | | | | 0x2, July 23 | 0x2, July 17 |
| Ghost Owl | | | | | 2x0, July 26 |

Table 6 shows, for each team, the number of pairs of games played on consecutive days according to the official schedule played in KCC2020, as displayed in Tables 4 and 5. These schedules were unbalanced and unfair, particularly for Group A. We observed that team Rox Gaming had three pairs of games on consecutive days and only one idle period of two days (July 20 and 21) for rest. On the other hand, all other teams had only one pair of games on two consecutive days, consequently with more preparation time between their games. Furthermore, neither group had

| Round number/ Position | Round 1 | Round 2 | Round 3 | Round 4 | Round 5 | Round 6 | Round 7 | Round 8 | Round 9 | Round 10 (final) | Round robin (result) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st place | Turnso Gaming 0.625 | QG Happy 0.766 | QG Happy 0.653 | Turnso Gaming 0.687 | Turnso Gaming 0.687 | Turnso Gaming 0.712 | QG Happy 0.661 | Turnso Gaming 0.691 | Turnso Gaming 0.720 | Turnso Gaming 0.714 | Turnso Gaming 4p |
| 2nd place | QG Happy 0.625 | eStar Pro 0.566 | Turnso Gaming 0.625 | QG Happy 0.666 | QG Happy 0.562 | QG Happy 0.612 | Turnso Gaming 0.651 | QG Happy 0.621 | QG Happy 0.625 | QG Happy 0.571 | QG Happy 3p |
| 3rd place | AG Super Play 0.625 | Turnso Gaming 0.500 | eStar Pro 0.596 | eStar Pro 0.645 | eStar Pro 0.562 | AG Super Play 0.575 | eStar Pro 0.581 | eStar Pro 0.586 | eStar Pro 0.529 | eStar Pro 0.571 | eStar Pro 3p |
| 4th place | eStar Pro 0.375 | AG Super Play 0.500 | AG Super Play 0.546 | AG Super Play 0.479 | AG Super Play 0.562 | eStar Pro 0.512 | AG Super Play 0.530 | AG Super Play 0.560 | AG Super Play 0.613 | AG Super Play 0.571 | AG Super Play 3p |
| 5th place | Team WE 0.375 | Team WE 0.433 | Team WE 0.375 | Team WE 0.333 | Team WE 0.437 | Team WE 0.412 | Team WE 0.429 | Team WE 0.399 | Team WE 0.375 | Team WE 0.428 | Team WE 2p |
| 6th place | Rox Gaming 0.375 | Rox Gaming 0.233 | Rox Gaming 0.203 | Rox Gaming 0.187 | Rox Gaming 0.187 | Rox Gaming 0.175 | Rox Gaming 0.146 | Rox Gaming 0.140 | Rox Gaming 0.136 | Rox Gaming 0.142 | Rox Gaming 0p |

Figure 1: Group A of KCC2020.

a compact schedule, not all teams played every day, and both schedules took longer than the minimum necessary days for a single round-robin tournament.

Table 6: Pairs of games on consecutive days played by each team.

| Group A | | Group B | |
|---|---|---|---|
| Team | Pairs of games on consecutive days | Team | Pairs of games on consecutive days |
| Turnso Gaming | 1 | Mighty Tiger | 2 |
| QG Happy | 1 | ViCi | 3 |
| AG Super Play | 1 | Douyu | 2 |
| eStar Pro | 1 | TTG X-Quest | 3 |
| Team WE | 1 | Ghost Owl | 3 |
| Rox Gaming | 3 | Esports of Macau | 3 |

The dynamic scheduling approach was applied to Groups A and B. All teams had the same ratings in the first round. Therefore, the first pairings were randomly arranged. The actual results in Tables 4 and 5 simulate the result of each game in the new schedule.

Figures 1 and 2 illustrate how the rankings of the six teams in Groups A and B, respectively, evolved along ten rounds scheduled by the dynamic approach, corresponding to twice the number of rounds in the round-robin tournament. Each cell displays the rating of the corresponding team and round (number of points for the round-robin column). Blocks of cells in the same round with a common border in bold correspond to teams tied with the same rating (or the same number of points). The results for Group B were very stable: round 3 has already obtained without ties the same ranking found in round 10, which is the same obtained by the full round-robin scheduling. For Group A, the situation was only slightly different. While the fifth and sixth places have been stable since the first round, there were alternations in the first four positions. The fourth (without ties) and fifth rounds have already obtained the same ranking found in round 10 (the same obtained by the full round-robin scheduling). However, some oscillations were still observed in rounds 6 and 7.

| Round number/ Position | Round 1 | Round 2 | Round 3 | Round 4 | Round 5 | Round 6 | Round 7 | Round 8 | Round 9 | Round 10 (final) | Round robin (result) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st place | Mighty Tiger Gaming 0.625 | Mighty Tiger Gaming 0.711 | Mighty Tiger Gaming 0.796 | Mighty Tiger Gaming 0.687 | Mighty Tiger Gaming 0.687 | Mighty Tiger Gaming 0.712 | Mighty Tiger Gaming 0.742 | Mighty Tiger Gaming 0.775 | Mighty Tiger Gaming 0.720 | Mighty Tiger Gaming 0.714 | Mighty Tiger Gaming 4p |
| 2nd place | ViCi Gaming 0.625 | ViCi Gaming 0.711 | ViCi Gaming 0.653 | ViCi Gaming 0.666 | ViCi Gaming 0.562 | ViCi Gaming 0.612 | ViCi Gaming 0.651 | ViCi Gaming 0.621 | ViCi Gaming 0.625 | ViCi Gaming 0.571 | ViCi Gaming 3p |
| 3rd place | Douyu Gaming 0.625 | Douyu Gaming 0.488 | Douyu Gaming 0.546 | Douyu Gaming 0.645 | Douyu Gaming 0.562 | Douyu Gaming 0.575 | Douyu Gaming 0.529 | Douyu Gaming 0.560 | Douyu Gaming 0.613 | Douyu Gaming 0.571 | Douyu Gaming 3p |
| 4th place | TTG X-Quest 0.375 | Ghost Owl Gaming 0.488 | TTG X-Quest 0.425 | TTG X-Quest 0.479 | TTG X-Quest 0.562 | TTG X-Quest 0.512 | TTG X-Quest 0.489 | TTG X-Quest 0.502 | TTG X-Quest 0.529 | TTG X-Quest 0.571 | TTG X-Quest 3p |
| 5th place | Ghost Owl Gaming 0.375 | TTG X-Quest 0.355 | Ghost Owl Gaming 0.403 | Ghost Owl Gaming 0.333 | Ghost Owl Gaming 0.437 | Ghost Owl Gaming 0.412 | Ghost Owl Gaming 0.438 | Ghost Owl Gaming 0.406 | Ghost Owl Gaming 0.375 | Ghost Owl Gaming 0.428 | Ghost Owl Gaming 2p |
| 6th place | Esports of Macau China 0.375 | Esports of Macau China 0.244 | Esports of Macau China 0.175 | Esports of Macau China 0.187 | Esports of Macau China 0.187 | Esports of Macau China 0.175 | Esports of Macau China 0.146 | Esports of Macau China 0.133 | Esports of Macau China 0.136 | Esports of Macau China 0.142 | Esports of Macau China 0p |

Figure 2: Group B of KCC2020.

These results show that the rankings become stable, and the outcome of tournaments played with the Swiss-system design is only slightly sensitive to the number of rounds played, provided that the tournament organizers warrant a sufficient number of rounds. The pairings obtained by the dynamic approach after five rounds are displayed in Tables 7 and 8.

Table 7: Pairings obtained by the dynamic scheduling approach for group A of KCC2020

| | Rounds | | | | |
|---|---|---|---|---|---|
| Teams | 1 | 2 | 3 | 4 | 5 |
| Turnso Gaming | Team WE | eStar Pro | QG Happy | AG Super Play | Rox Gaming |
| QG Happy | eStar Pro | AG Super Play | Turnso Gaming | Rox Gaming | Team WE |
| AG Super Play | Rox Gaming | QG Happy | Team WE | Turnso Gaming | eStar Pro |
| eStar Pro | QG Happy | Turnso Gaming | Rox Gaming | Team WE | AG Super Play |
| Team WE | Turnso Gaming | Rox Gaming | AG Super Play | eStar Pro | QG Happy |
| Rox Gaming | AG Super Play | Team WE | eStar Pro | QG Happy | Turnso Gaming |

Table 8: Pairings obtained by the dynamic scheduling approach for group B of KCC2020

| | Rounds | | | | |
|---|---|---|---|---|---|
| Teams | 1 | 2 | 3 | 4 | 5 |
| Mighty Tiger | Ghost Owl | TTG X-Quest | ViCi | Douyu | Esports Macau |
| ViCi | TTG X-Quest | Douyu | Mighty Tiger | Esports Macau | Ghost Owl |
| Douyu | Esports Macau | ViCi | Ghost Owl | Mighty Tiger | TTG X-Quest |
| TTG X-Quest | ViCi | Mighty Tiger | Esports Macau | Ghost Owl | Douyu |
| Ghost Owl | Mighty Tiger | Esports Macau | Douyu | TTG X-Quest | ViCi |
| Esports of Macau | Douyu | Ghost Owl | TTG X-Quest | ViCi | Mighty Tiger |

We performed an additional experiment to illustrate this issue, considering a tournament with $n = 6$ teams A, B, C, D, E, and F. We assume that A wins B, B wins C, C wins D, D wins E, E wins F, and the transitivity relation holds (i.e., if the team $i$ wins team $j$ and the team $j$ wins team $k$, then team $i$ wins team $k$). Figure 3 illustrates how the rankings of the six teams evolve along five rounds scheduled by the dynamic approach and compares the final ranking with that obtained by the single round-robin tournament. These results show that the final ranking can be quickly obtained with a few rounds if transitivity holds (although this property rarely, or never, holds in practice).

| Round number/ Position | Round 1 | Round 2 | Round 3 | Round 4 | Round 5 (final) | Round robin (result) |
|---|---|---|---|---|---|---|
| 1st place | A 0.625 | A 0.711 | A 0.796 | A 0.833 | A 0.812 | A 5p |
| 2nd place | B 0.625 | B 0.711 | B 0.653 | B 0.666 | B 0.687 | B 4p |
| 3rd place | C 0.625 | C 0.488 | C 0.546 | C 0.500 | C 0.562 | C 3p |
| 4th place | D 0.375 | E 0.488 | D 0.425 | D 0.500 | D 0.437 | D 2p |
| 5th place | E 0.375 | D 0.355 | E 0.403 | E 0.333 | E 0.312 | E 1p |
| 6th place | F 0.375 | F 0.244 | F 0.175 | F 0.166 | F 0.187 | F 0p |

Figure 3: Schedule under transitivity winning relations.

### *5.2. Results for randomly generated larger tournaments*

This section applies the dynamic scheduling approach to larger tournaments with up to $n = 80$ teams. We evaluate its computational efficiency and assess the consistency of the numerical results. We consider two model parameters in our computational study:

- $L$ is the minimum number of rounds between two consecutive games of any pair of teams. $L = 0$ corresponds to the unconstrained case, while for $L \geq R - 1$ no repetitions are allowed.

- $D$ is the maximum difference between the time slots of the games played by any team in any pair of consecutive rounds. $D \geq n/2 - 1$ corresponds to the unconstrained case, while for $D = 1$, the position of the game played by each team may change by at most one unit between any two consecutive rounds.

We also evaluate and discuss the sensitivity of the final ranking obtained by the dynamic scheduling approach concerning the number of rounds $R$ externally set by the tournament organizers.

The winning probabilities used to randomly determine the winner of each game along the simulation of the tournament were calculated as follows. After a round terminates and before the games of a new round are played, all team ratings are updated using Colley's method, and the teams $i_{min}$ and $i_{max}$ with the minimum and maximum ratings, respectively, are determined: $i_{min} = \text{argmin}\{r_i : i = 1, \ldots, n\}$ and $i_{max} = \text{argmax}\{r_i : i = 1, \ldots, n\}$. Furthermore, let $\Delta = r_{i_{max}} - r_{i_{min}}$ be the maximum difference between the current ratings of all pairs of teams. For every game scheduled to be played between teams $i$ and $j$, the winning probabilities of teams $i$ and $j$

are calculated as follows:
(1) If $r_i = r_j$, then the winning probability of each team $i$ and $j$ is set to 0.5.
(2) If $r_i = r_{i_{max}}$ and $r_j = r_{i_{min}}$, then the winning probability of team $i$ is set to a simulation parameter $p_V$ and that of team $j$ is set to $1 - p_V$.
(3) Otherwise, assume $r_i > r_j$, let $\delta = r_i - r_j$, and set the winning probability of team $i$ to $(\delta/\Delta) \cdot (p_V - 0.5) + 0.5$ and that of team $j$ to $0.5 - (\delta/\Delta) \cdot (p_V - 0.5)$.
The maximum winning probability $p_V$ of team $i$ over team $j$, considering that their ratings satisfy $r_i \geq r_j$, belongs to the interval [0.5,1.0].

Simulations were performed for $n$ = 16, 20, 30, 40, 60, 80, and 100 teams, for the maximum winning probability $p_V = 0.6$ and 0.8, and for the model parameters $L = 1, \ldots, 10$ and $D = 1, \ldots, 15$.

Figure 4 displays the average execution times per round of the dynamic scheduling approach for $n/2$ rounds as the number of teams ranges from 16 to 80. We observe that the approach is feasible in practice and the execution times are small. The execution times increase for larger values of $L$ and $D$. Furthermore, comparing the plots in Figures 4(a) and (b), we notice that the maximum winning probability $p_V$ does not significantly influence the execution times.

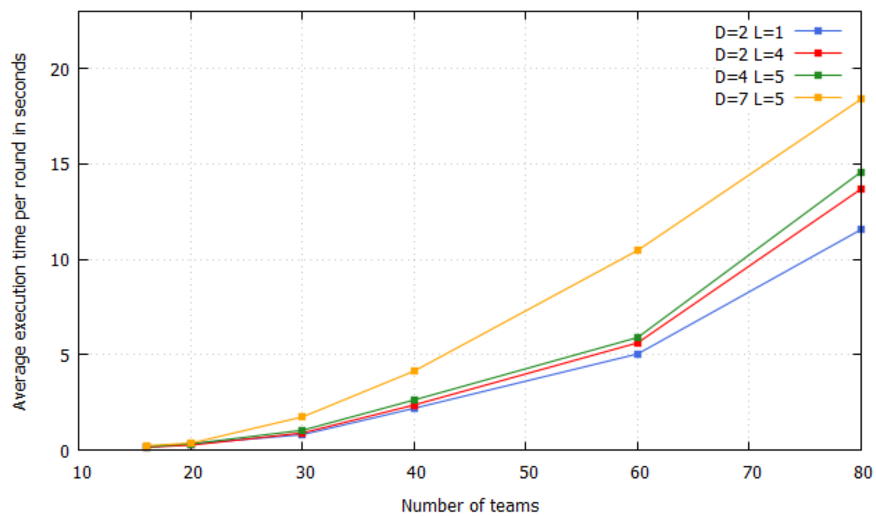
The influence of parameters $L$ and $D$ in the execution times is better illustrated in Figure 5. Figure 5(a) shows that the execution times become more significant as the distance parameter $D$ increases, while Figure 5(b) shows that there is less variation with the repetition parameter $L$. This behavior is compatible with the shape of the surface illustrated in the 3D representation displayed by Figure 5(c). We observe that the larger (resp. smaller) the value of $L$ (resp. $D$), the more constrained the scheduling problem solved in each round is. In practice, large values of $L$ (to avoid repetitions of the same game) and $D \approx 2$ are adequate.

Figures 6(a) and 6(b) illustrate for two tournaments with $n = 16$ and $n = 20$, respectively, how the teams ranked in the first six positions evolve from round $\lceil log_2 n \rceil$ to round $n - 1$. They show that the teams ranked in the first and second positions do not change along all these rounds (i.e., after the first $\lceil log_2 n \rceil$ rounds), although some changes occur between the third and sixth positions. The rating, pairing, and scheduling methods seem very appropriate to lead to the tournament's winner.

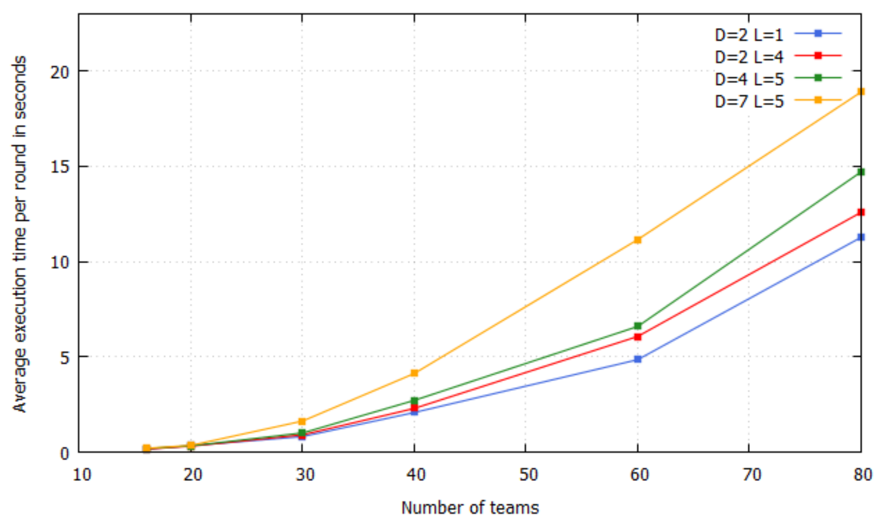
### *5.3. Implications of using dynamic scheduling with the Swiss system*

The results and analyses in the previous sections showed that the proposed dynamic scheduling method for e-sports has some advantages over other, more traditional tournament designs and scheduling algorithms. First, it allows tournaments with more participants than round-robin tournaments. Second, it requires fewer rounds to define the champion and the runner-up. Third, the dynamic approach can maximize the attractiveness of the games scheduled in each round, leading to higher attendance and ticket sales. In addition, games between teams with similar ratings increase the competitive balance of the tournament and make the competition more attractive for the viewers, thus leading to greater profits for the organizers. Greater pool prizes for the participants and increased exposure for advertisers and sponsors are also natural consequences that may lead to the advancement of e-sports.

The simultaneous optimization of team pairing (opponent selection) and game scheduling (assignment of time-slots) in the same model makes the scheduling procedure much more straightforward. The numerical results showed that the proposed dynamic scheduling method runs very fast in practice once the parameters are set. The optimization model is compatible with other ranking methods and sports, which makes it a general framework suitable for different situations. For example, if traveling is not a key issue for the tournament (or if there is enough time to organize




(a) Execution times for $R = n/2$ rounds and the maximum winning probability $p_V = 0.6$.




(b) Execution times for $R = n/2$ rounds and the maximum winning probability $p_V = 0.8$.

Figure 4: Average execution times per round with the increase in the number of teams.

the logistics between two consecutive rounds), then there is no need to fix the opponents in each game beforehand. In this case, the organizer can use dynamic scheduling to improve both the suspense and the fairness of the competition. Thus the proposed method for e-sports can also be extended to traditional sports with similar properties.

This change is the case of the European Champions League, which has decided to use the Swiss system for the 2024-25 season. The number of teams will be raised from 32 to 36. Teams are guaranteed a minimum of 10 games with the new design. In consequence, the tournament almost doubles in size, from a total of 125 games to 225. The top teams should be able to play more meaningful games against their main rivals. More games equals more money, and the biggest teams always take a larger slice of the broadcasting revenues (ESPN, 2021).

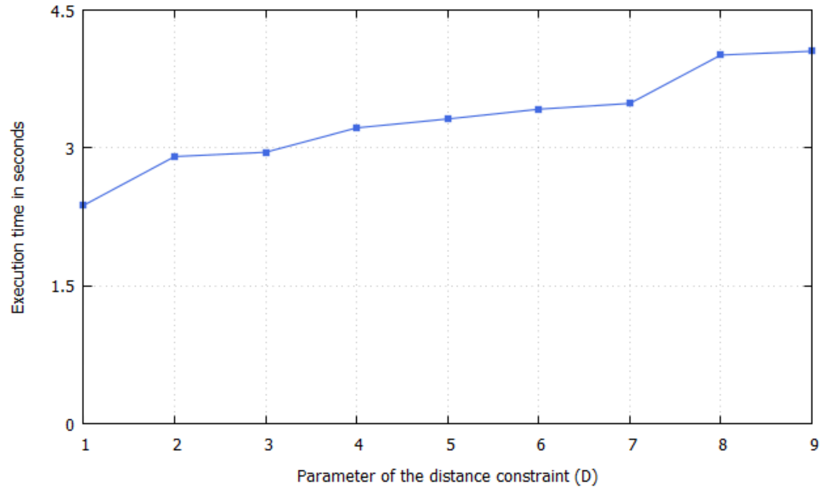
## 6. Concluding remarks

Electronic sports have seen fast growth in market value and number of participants in recent years. At the same time, e-sports generate more revenues and become official sports in events like the Asian Games. Tournaments of e-sports attract progressively more attention and pay higher total prize pools.

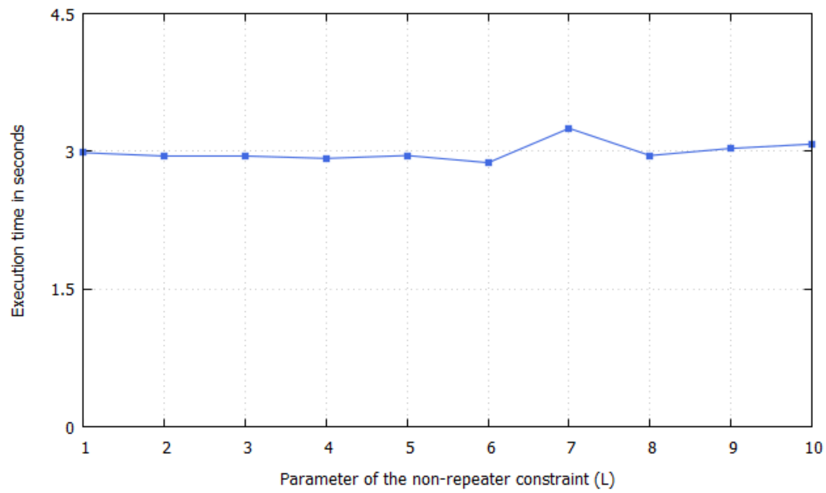
Although tournament scheduling plays a vital role in traditional sports and the optimization of schedules significantly improves their fairness and logistics, more needs to be done regarding e-sports tournaments. We presented in this work the first application of optimization methods to the scheduling of e-sports tournaments. The main methodological contribution of this article is a new dynamic scheduling strategy for e-sports tournaments based on the Swiss system design. Colley's method is used for updating the ratings of all competitors. These ratings are used for maximizing game attractiveness and viewers' utility in the integer programming model of the team pairing and game scheduling problem solved in each round. The two parameters used in the optimization model are related to fairness and attractiveness, giving more flexibility to the tournament organizers.

The dynamic approach was validated with real-life data collected from the 2020 Honor of Kings World Champion Cup (KCC2020) and using real-size randomly generated test problems with up to 80 competitors. The integer programming problem in each round can be solved fast, independently of the parameter values set by the tournament organizers. The numerical results show that convergence and stability can be obtained in $\lceil \log_2 n \rceil$ or a few more rounds, where $n$ is the number of competitors.

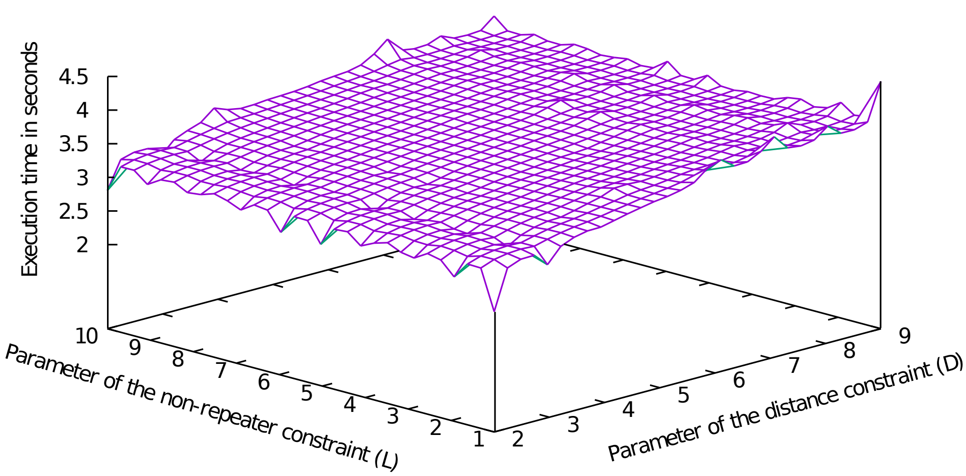
We only considered using Colley's ratings for dynamic scheduling. We leave for future work the discussion about other rating methods that do not affect the optimization strategy proposed here. We also plan to incorporate the teams' subjective efforts to calculate the game attractiveness, which makes the approach more realistic while not increasing the difficulty of the optimization models. Investigating how the prize allocation distribution would affect the scheduling results is also interesting.




(a) Execution times for $n = 20$ teams, $R = 10$ rounds, $L = 2$, and $p_V = 0.8$.




(b) Execution times for $n = 20$ teams, $R = 10$ rounds, $D = 3$, and $p_V = 0.8$.




(c) 3-D view of the execution times for $n = 20$ teams, $R = 10$ rounds, and $p_V = 0.8$.

Figure 5: Variation of the execution times with the parameters $L$ and $D$.

| Round number/ Position | Round 4 | Round 5 | Round 6 | Round 7 | Round 8 | Round 9 | Round 10 | Round 11 | Round 12 | Round 13 | Round 14 | Round 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st place | Team 16 0.930 | Team 16 0.980 | Team 16 0.989 | Team 16 0.933 | Team 16 0.948 | Team 16 0.938 | Team 16 0.863 | Team 16 0.885 | Team 16 0.909 | Team 16 0.932 | Team 16 0.932 | Team 16 0.889 |
| 2nd place | Team 1 0.759 | Team 1 0.817 | Team 1 0.820 | Team 1 0.866 | Team 1 0.891 | Team 1 0.908 | Team 1 0.847 | Team 1 0.811 | Team 1 0.780 | Team 1 0.795 | Team 1 0.823 | Team 1 0.777 |
| 3rd place | Team 15 0.670 | Team 2 0.686 | Team 14 0.708 | Team 14 0.760 | Team 14 0.729 | Team 14 0.691 | Team 14 0.711 | Team 13 0.738 | Team 14 0.765 | Team 14 0.746 | Team 14 0.714 | Team 14 0.729 |
| 4th place | Team 5 0.662 | Team 14 0.627 | Team 2 0.629 | Team 15 0.663 | Team 15 0.626 | Team 4 0.635 | Team 4 0.710 | Team 14 0.732 | Team 13 0.700 | Team 13 0.657 | Team 13 0.682 | Team 13 0.720 |
| 5th place | Team 12 0.608 | Team 15 0.618 | Team 15 0.609 | Team 2 0.575 | Team 13 0.593 | Team 13 0.631 | Team 13 0.690 | Team 4 0.688 | Team 4 0.648 | Team 4 0.616 | Team 7 0.593 | Team 4 0.606 |
| 6th place | Team 2 0.590 | Team 5 0.614 | Team 12 0.576 | Team 3 0.559 | Team 4 0.592 | Team 15 0.574 | Team 15 0.535 | Team 2 0.545 | Team 3 0.548 | Team 3 0.585 | Team 4 0.588 | Team 3 0.600 |

(a) Teams ranked in the first six positions for $n = 16$, $D = 2$, $L = 4$, and $p_V = 0.8$.

| Round number/ Position | Round 5 | Round 6 | Round 7 | Round 8 | Round 9 | Round 10 | Round 11 | Round 12 | Round 13 | Round 14 | Round 15 | Round 16 | Round 17 | Round 18 | Round 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st place | T.14 0.945 | T.14 0.864 | T.14 0.904 | T.14 0.934 | T.14 0.952 | T.14 0.942 | T.14 0.909 | T.14 0.928 | T.14 0.944 | T.14 0.901 | T.14 0.856 | T.14 0.880 | T.14 0.901 | T.14 0.922 | T.14 0.933 |
| 2nd place | T.16 0.751 | T.16 0.823 | T.16 0.860 | T.16 0.803 | T.16 0.840 | T.16 0.848 | T.16 0.894 | T.16 0.911 | T.16 0.927 | T.16 0.868 | T.16 0.834 | T.16 0.812 | T.16 0.836 | T.16 0.857 | T.16 0.868 |
| 3rd place | T.5 0.737 | T.5 0.811 | T.19 0.778 | T.5 0.801 | T.5 0.754 | T. 5 0.689 | T.1 0.706 | T.19 0.729 | T.19 0.752 | T.19 0.777 | T.19 0.797 | T.19 0.771 | T.19 0.795 | T.19 0.819 | T.19 0.806 |
| 4th place | T.12 0.721 | T.19 0.746 | T.5 0.749 | T.19 0.744 | T.19 0.722 | T.19 0.671 | T.19 0.703 | T.1 0.678 | T.1 0.654 | T.5 0.657 | T.5 0.675 | T.5 0.707 | T.5 0.702 | T.5 0.727 | T.5 0.741 |
| 5th place | T.19 0.710 | T.7 0.681 | T.7 0.630 | T.12 0.665 | T.1 0.681 | T.1 0.658 | T.10 0.681 | T.5 0.645 | T.10 0.624 | T.10 0.652 | T.13 0.666 | T.17 0.676 | T.17 0.655 | T.17 0.643 | T.1 0.620 |
| 6th place | T.7 0.634 | T.12 0.661 | T.12 0.603 | T.1 0.625 | T.12 0.635 | T.10 0.650 | T.5 0.666 | T.10 0.644 | T.5 0.616 | T.13 0.632 | T.17 0.645 | T.13 0.642 | T.13 0.625 | T.13 0.610 | T.17 0.617 |

(b) Teams ranked in the first six positions for $n = 20$, $D = 2$, $L = 4$, and $p_V = 0.8$.

Figure 6: Teams ranked in the first six positions from round $\lceil log_2 n \rceil$ to round $n - 1$.

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