On General Model-Checkers

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Proposal

Goal Specification and Construction of Model-checkers

General graphical approach

- General model-checker instantiated to specific model-checker

Two-component approach:

1. Logic-dependent converter — graphical description of specific logic's semantics (user-provided)
2. General model checker — no user's concern

Applications: teaching; exploration of developing logics

Outline

1. Basic Ideas
2. Modal Languages & Models
3. Graphical Concepts & Constructions
4. Semantics for Graphical Concepts
5. Graphical Representation of Semantics
6. Examples

Classical Modal Logic

Intuitionistic Modal Logic

7. General Converter

8. Instantiated graphical converters

for Classical Modal Logic

for Intuitionistic Modal Logic

9. General graphical model-checker

10. Concluding Remarks

comments, extensions

1 Basic ideas

Model checker one for each logic

$$\text{Formula } \varphi \longrightarrow \boxed{\text{Model-checker}} \longrightarrow \left\{ \begin{array}{c} \text{Yes} \\ \text{No} \end{array} \right\} \begin{array}{l} \text{if } \mathfrak{M}, a \Vdash \varphi \\ \text{if } \mathfrak{M}, a \nVdash \varphi \end{array}$$

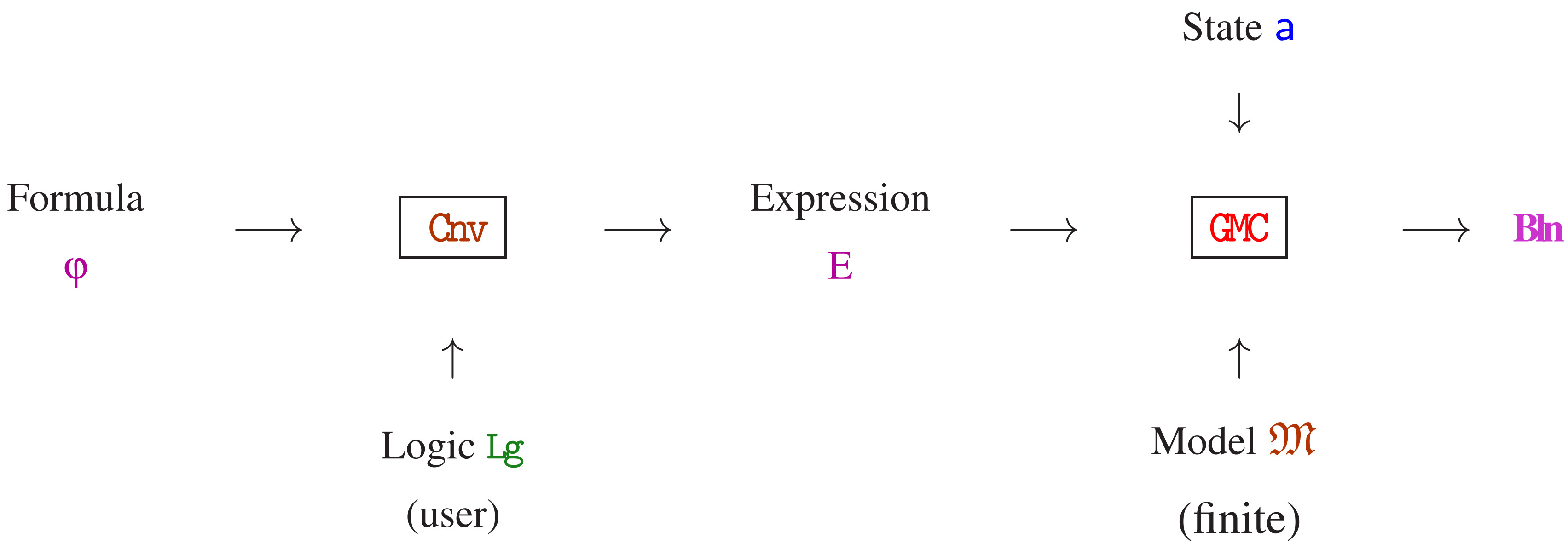
$$\uparrow$$

Model $\mathfrak{M}$

(finite)

state a

General model-checker GMC with converter Cnv



Graphical Converter

Formula φ $\longrightarrow$ [$\mathtt{Cnv}$] $\longrightarrow$ Graphical expression E

$\uparrow$

Logic $\mathtt{Lg}$

receives a formula & converts it to a graphical expression :

eliminating symbols of the formula according to the graphical description of the semantics given by user

Modal Languages

$$\mathsf{ML} \quad \begin{cases} \mathbb{PL} & \textit{propositional letters} \\ \dagger & \textit{0-ary connectives} \\ \nabla & \textit{1-ary connectives} \\ \bullet & \textit{2-ary connectives} \\ \mu \in \Xi & \textit{modalities } \Xi \end{cases}$$

Formulas Φ set of formulas

$$\varphi \;::=\; p \mid \dagger \mid \nabla\varphi \mid \varphi' \bullet \varphi'' \mid \mu\varphi \qquad (p \in \mathbb{PL}) \qquad (1)$$

Model $\mathfrak{M}$

Universe $\mathsf{M} \neq \emptyset$

$$\begin{cases} \text{subset } \varphi^{\mathfrak{M}} \subseteq \mathsf{M} & \varphi \in \Phi \\ \text{2-ary relation } \mu^{\mathfrak{M}} \text{ on } \mathsf{M} & \mu \in \Xi \end{cases}$$

Graphical Representation of Semantics

1. Representing — formula at state

   φ — formula φ
   w — does hold at w

   $\overline{\varphi}$ — formula φ
   w — does <u>not</u> hold at w

   ¡ — $\overline{\varphi}$ expression

2. Accessibility between states — $u \xrightarrow{r} v$

3. Evaluation point — $\widehat{x}$ — distinguished state

2 Graphical Concepts & Constructions

		Intuitive meaning
Node names Nd	finite set	states
Arcs	1-ary	2-ary
unary	$w \dashrightarrow E$	w pertains to E
binary	$u \xrightarrow{L} v$	accessibility between states
(D) Draft	finite sets of nodes & arcs	restriction on states ($\wedge$)
(P) Page	draft & distinguished node	set of states ($\exists$)
(B) Book	finite set of pages	set of states ($\vee$)
(E) Expression		
	formulas pages books & their complements	set of states

(PG) *Page of expression set* $\mathrm{PG}(F) \coloneqq \begin{array}{l} \hat{x} - - - \!\!\prec F_1 \\ \quad\vdots \\ \hat{x} - - - \!\!\prec F_h \end{array}$ $\quad \mathrm{Pg}(E) \coloneqq \mathrm{PG}(\{E\})$

(BK) *Book of expression set*

$$\mathrm{BK}[F] \coloneqq \left\{ \begin{array}{c} \mathrm{Pg}(F_1) \\ \vdots \\ \mathrm{Pg}(F_h) \end{array} \right\} = \left\{ \begin{array}{c} \hat{x} - - -\!\!\prec F_1 \\ \vdots \\ \hat{x} - - -\!\!\prec F_h \end{array} \right\}$$

(FP) *Follow-page* $(\mu \in \Xi)$ $\mathrm{FP}[\mu, F) \coloneqq \hat{x} \xrightarrow{\mu} y \begin{array}{l} - - - - \!\!\prec F_1 \\ \quad\vdots \\ - - - - \!\!\prec F_h \end{array}$

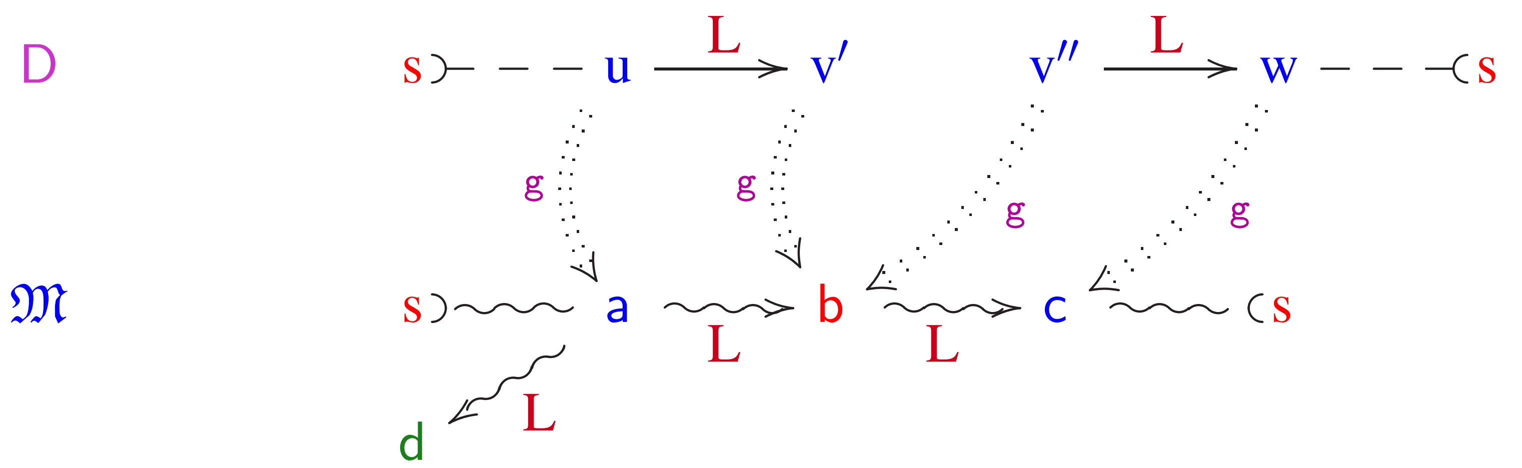
(fp) $\mathrm{fp}[r, E) \coloneqq \mathrm{FP}[\langle r \rangle, \{E\}) = \hat{x} \xrightarrow{r} y - - -\!\!\prec E$

3 Semantics for Graphical Concepts

Model $\mathfrak{M}$ over universe M

Exmpl. Draft satisfaction & page behavior

Assignment g $\quad u \mapsto \mathsf{a}; \quad v', v'' \mapsto \mathsf{b}; \quad w \mapsto \mathsf{c}$



Assignment g satisfies draft D

(E) *Expression*: set of expression : $(\mathrm{E})_{\mathfrak{M}} \subseteq \mathsf{M}$

- formula $\varphi \in \Phi$ $(\varphi)_{\mathfrak{M}} \;\coloneqq\; \varphi^{\mathfrak{M}}$
- expression E (page or book) $(\mathrm{E})_{\mathfrak{M}} \;\coloneqq\; [\![\mathrm{E}]\!]_{\mathfrak{M}}$
- complemented expression $\overline{\mathrm{E}}$ $(\overline{\mathrm{E}})_{\mathfrak{M}} \;\coloneqq\; \mathsf{M} \setminus (\mathrm{E})_{\mathfrak{M}}$

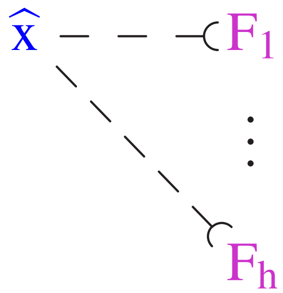
(P) *Page* $\mathsf{P} = (\mathsf{D} : \mathrm{u})$ behavior $[\![\mathsf{P}]\!]_{\mathfrak{M}}$

Exmpl.: state $\mathsf{a} = \mathrm{u}^{\mathrm{g}}$ $\therefore$ $\mathsf{a} \in [\![\mathsf{P}]\!]_{\mathfrak{M}}$

(B) *Book* B behavior $[\![\mathsf{B}]\!]_{\mathfrak{M}} \coloneqq \bigcup_{\mathsf{P} \in \mathsf{B}} [\![\mathsf{P}]\!]_{\mathfrak{M}}$

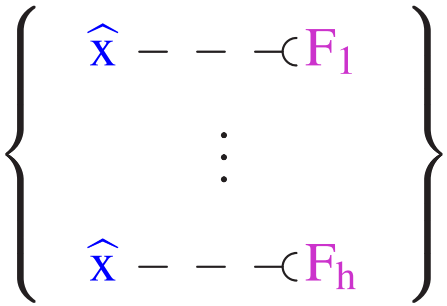
(cf. Graphical constructs, p. 11)

(PG) $\mathrm{PG}(F) =$



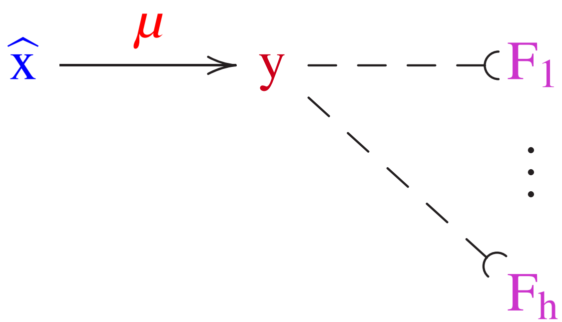
$[\![\mathrm{PG}(F)]\!]_{\mathfrak{M}} \coloneqq (\mathrm{F}_1)_{\mathfrak{M}} \cap \cdots \cap (\mathrm{F}_h)_{\mathfrak{M}}$

(BK) $\mathrm{BK}[F] =$



$[\![\mathrm{BK}[F]]\!]_{\mathfrak{M}} \coloneqq (\mathrm{F}_1)_{\mathfrak{M}} \cup \cdots \cup (\mathrm{F}_h)_{\mathfrak{M}}$

(FP) $\mathrm{FP}[\mu, F) =$



$$[\![\mathrm{FP}[\mu, F)]\!]_{\mathfrak{M}} = \{\mathsf{a} \in \mathsf{M} / \exists\, \mathsf{b} \in \mathsf{M}\,[\,(\mathsf{a}, \mathsf{b}) \in \mu^{\mathfrak{M}} \wedge \mathsf{b} \in [\![\mathrm{PG}(F)]\!]_{\mathfrak{M}}\,]\}$$

4 Graphical Formulations of Semantics

user's task

1. Translate satisfaction clauses to graphical expressions
2. Formulate resulting expressions by means of expression constructs:

   simultaneous PG, alternatives BK and change FP

Simple Modal languages

$\dagger := \bot$ $\quad\quad\quad \nabla := \neg$ $\quad\quad\quad \bullet \in \{\wedge, \vee, \rightarrow\}$

$\Xi := \{\langle r \rangle / r \in RN\} \cup \{[r] / r \in RN\}$ $\quad\quad\quad$ RN relation names

Model $\mathfrak{M}$

Universe $M \neq \emptyset$

$$\begin{cases} \text{subset } p^{\mathfrak{M}} \subseteq M & p \in PL \\ \text{2-ary relation } r^{\mathfrak{M}} \text{ on } M & r \in RN \end{cases}$$

Exmpl. Classical modal semantics — Graphical KM semantics

(p) $\mathfrak{C}, \mathsf{a} \Vdash p$ iff $\mathsf{a} \in p^{\mathfrak{C}}$

(⊥) $\mathfrak{C}, \mathsf{a} \nVdash \bot$ iff $\mathsf{a} \in (\{\})_{\mathfrak{C}}$ empty book

(¬) $\mathfrak{C}, \mathsf{a} \Vdash \neg\varphi$ iff $\mathfrak{C}, \mathsf{a} \nVdash \varphi$ iff $\mathsf{a} \in (\overline{\varphi})_{\mathfrak{C}}$

complemented expression

(∧) $\mathfrak{C}, \mathsf{a} \Vdash \psi \wedge \theta$ iff $\mathfrak{C}, \mathsf{a} \Vdash \psi$ & $\mathfrak{C}, \mathsf{a} \Vdash \theta$ iff

$\mathsf{a} \in (\,\psi \succ\!- - - \widehat{x} - - -\!\prec \theta\,)_{\mathfrak{C}}$ 1-node page

(∨) $\mathfrak{C}, \mathsf{a} \Vdash \psi \vee \theta$ iff $\mathfrak{C}, \mathsf{a} \Vdash \psi$ or $\mathfrak{C}, \mathsf{a} \Vdash \theta$

$$\mathsf{a} \in \left(\left\{ \begin{array}{l} \psi \succ\!- - - \widehat{x} \\ \widehat{x} - - -\!\prec \theta \end{array} \right\} \right)_{\mathfrak{C}}$$ 2-page book

$(\to)$ $\mathfrak{C}, a \Vdash \psi \to \theta$ iff $\mathfrak{C}, a \nVdash \psi$ or $\mathfrak{C}, a \Vdash \theta$ iff

$$a \in \left(\left\{ \begin{array}{c} \overline{\psi} \succ - - - \widehat{x} \\ \\ \widehat{x} - - - \prec \theta \end{array} \right\} \right)_{\mathfrak{C}}$$

2-page book

$(\langle\rangle)$ $\mathfrak{C}, a \Vdash \langle r \rangle \varphi$ iff for some b with $(a, b) \in r^{\mathfrak{C}}$: $\mathfrak{C}, b \Vdash \varphi$ iff

$$a \in (\widehat{x} \xrightarrow{r} y - - - \prec \varphi)_{\mathfrak{C}}$$

2-node page

$([])$ $\mathfrak{C}, a \Vdash [r] \varphi$ iff for every b with $(a, b) \in r^{\mathfrak{C}}$: $\mathfrak{C}, b \Vdash \varphi$

iff $\nexists\, b\, ((a, b) \in r^{\mathfrak{C}}$ & $\mathfrak{C}, b \nVdash \varphi)$

iff

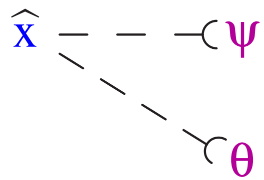
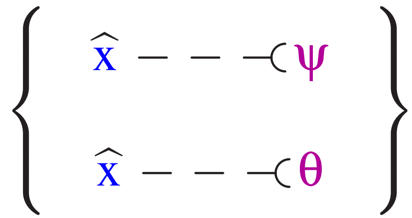
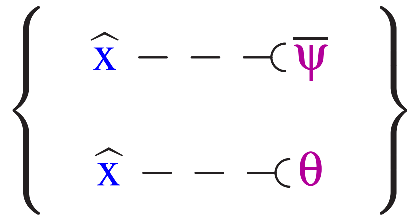
$$a \in \overline{(\widehat{x} \xrightarrow{r} y - - - \prec \overline{\varphi})}_{\mathfrak{C}}$$

complemented 2-node page

Formula	Graphical Expression	Construct
$\bot$	$\{\}$	$\mathsf{BK}[\emptyset]$
$\neg\varphi$	$\overline{\varphi}$	$\overline{\varphi}$
$\psi \land \theta$	$\psi \succ - - - \widehat{x} - - - \prec \theta$	$\mathsf{PG}(\{\psi, \theta\})$
$\psi \lor \theta$	$\left\{ \begin{array}{l} \widehat{x} - - \prec \psi \\ \widehat{x} - - \prec \theta \end{array} \right\}$	$\mathsf{BK}[\{\psi, \theta\}]$
$\psi \to \theta$	$\left\{ \begin{array}{l} \widehat{x} - - \prec \overline{\psi} \\ \widehat{x} - - \prec \theta \end{array} \right\}$	$\mathsf{BK}[\{\overline{\psi}, \theta\}]$
$\langle r \rangle \varphi$	$\widehat{x} \xrightarrow{r} y - - \prec \varphi$	$\mathsf{fp}[r, \varphi)$
$[r]\varphi$	$\overline{\widehat{x} \xrightarrow{r} y - - \prec \overline{\varphi}}$	$\overline{\mathsf{fp}[r, \overline{\varphi})}$

⇒ Elimination rules for the classical operators (cf. p. 18)

1. Propositional

$\vec{\bot}_{\mathtt{KM}}$	$:=$	$\mathsf{BK}[\emptyset]$	empty book $\{\ \}$
$\vec{\neg}_{\mathtt{KM}}[\varphi]$	$:=$	$\overline{\varphi}$	expression $\overline{\varphi}$
$\vec{\wedge}_{\mathtt{KM}}[\psi,\theta]$	$:=$	$\mathsf{PG}(\{\psi,\theta\})$	1-node page 
$\vec{\vee}_{\mathtt{KM}}[\psi,\theta]$	$:=$	$\mathsf{BK}[\{\psi,\theta\}]$	2-page book 
$\vec{\rightarrow}_{\mathtt{KM}}[\psi,\theta]$	$:=$	$\mathsf{BK}[\{\overline{\psi},\theta\}]$	2-page book 

2. Modalities

$\langle \vec{r} \rangle_{\mathtt{KM}}[\varphi]$	$:=$	$\mathrm{fp}[r, \varphi)$	follow-page	$\widehat{x} \xrightarrow{\ r\ } y \ - - -\!\!\multimap \varphi$
$[\vec{r}]_{\mathtt{KM}}[\varphi]$	$:=$	$\overline{\mathrm{fp}[r, \overline{\varphi})}$	complemented follow-page	$\overline{\widehat{x} \xrightarrow{\ r\ } y \ - - -\!\!\multimap \overline{\varphi}}$

Graphical specification for classical modal logic KM (cf. p. 19)

1. Propositional fragment: KM-elimination rules for $\bot, \neg, \wedge, \vee, \rightarrow$

$$\bot \coloneqq \{\ \} \qquad \neg\varphi \coloneqq \overline{\varphi} \qquad \psi \wedge \theta \coloneqq \mathsf{PG}(\{\psi, \theta\})$$

$$\psi \vee \theta \coloneqq \mathsf{BK}[\{\psi, \theta\}] \qquad \psi \rightarrow \theta \coloneqq \mathsf{BK}[\{\overline{\psi}, \theta\}]$$

2. Modal fragment: KM-elimination rules for $\langle\,\rangle, [\,]$

$$\langle r \rangle\, \varphi \coloneqq \mathsf{fp}[r, \varphi) \qquad [r]\, \varphi \coloneqq \overline{\mathsf{fp}[r, \overline{\varphi})}$$

This will give a graphical converter for KM (see p. 31)

Exmpl. Intuitionistic modal model $\mathfrak{I}$ with world precedence $\preceq^{\mathfrak{I}}$

Intuitionist modal semantics — Simpson

($\star$) For $p, \bot, \wedge, \vee$ and $\langle\,\rangle$: as in classical modal semantics (cf. p. 16).

($\neg$) $\mathfrak{I}, a \Vdash \neg\varphi$ iff for every b with $(a, b) \in \preceq^{\mathfrak{I}}$: $\mathfrak{I}, b \nVdash \varphi$
iff $\nexists\, b$ s.t. $(a, b) \in \preceq^{\mathfrak{I}}$ and $\mathfrak{I}, b \Vdash \varphi$

Graphical $\mathbb{JM}$ formulation: $\neg\varphi$ $\mathbb{JM}$-equivalent to

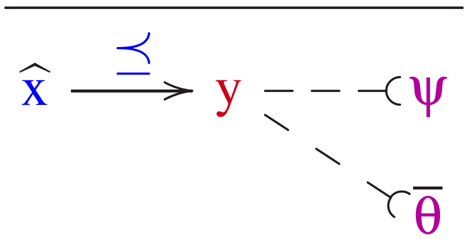
$$\hat{x} \xrightarrow{\preceq} y \;- - -\!\!\prec \varphi$$

complemented 2-node page

$(\to)$ $\mathfrak{I}, a \Vdash \psi \to \theta$ iff for all $(a, b) \in \preceq^{\mathfrak{I}}$: if $\mathfrak{I}, b \Vdash \psi$ then $\mathfrak{I}, b \Vdash \theta$

iff $\nexists$ b with $(a, b) \in \preceq^{\mathfrak{I}}$ s. t. $\mathfrak{I}, b \Vdash \psi$ & $\mathfrak{I}, b \nVdash \theta$

Graphical $\mathbb{JM}$ formulation: $\psi \to \theta$ $\mathbb{JM}$-equivalent to



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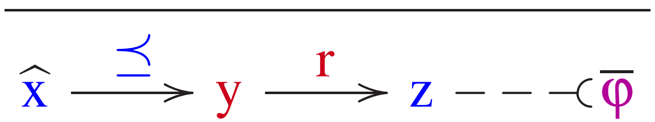
$([\,])$ $\mathfrak{I}, a \Vdash [r]\varphi$ iff whenever $(a, b) \in \preceq^{\mathfrak{I}}$ and $(b, c) \in r^{\mathfrak{I}}$: $\mathfrak{I}, c \Vdash \varphi$; i. e.

$\nexists$ b, c with $(a, b) \in \preceq^{\mathfrak{I}}$, $(b, c) \in r^{\mathfrak{I}}$ and $\mathfrak{I}, c \nVdash \varphi$

Graphical $\mathbb{JM}$ formulation: $[r]\varphi$ $\mathbb{JM}$-equivalent to



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Elimination rules for intuitionistic $\neg$, $\rightarrow$ and $[\,]$ (cf. p. 22)

$\vec{\neg}_{\mathtt{IP}}[\varphi]$	$\coloneqq$	$\overline{\mathrm{fp}[\preceq, \varphi)}$	complemented follow-page
$\vec{\rightarrow}_{\mathtt{IP}}[\psi, \theta]$	$\coloneqq$	$\overline{\mathrm{fp}[\preceq, \mathrm{PG}(\{\psi, \overline{\theta}\}))}$	complemented follow-page
$[\vec{\mathrm{r}}]_{\mathtt{JM}}[\varphi]$	$\coloneqq$	$\overline{\mathrm{fp}[\preceq, (\mathrm{fp}[\mathrm{r}, \overline{\varphi})))}$	complemented follow-page

Graphical specification for intuitionistic modal logic $\mathbb{JM}$ (cf. p. 24)

1. Propositional fragment:

   (a) $\mathbb{JM}$-elimination rules for $\bot, \wedge, \vee$

   $$\bot \coloneqq \{\ \} \qquad \psi \wedge \theta \coloneqq \mathsf{PG}(\{\psi, \theta\}) \qquad \psi \vee \theta \coloneqq \mathsf{BK}[\{\psi, \theta\}]$$

   (b) $\mathbb{JM}$-elimination rules for $\neg, \rightarrow$

   $$\neg \varphi \coloneqq \overline{\mathsf{fp}[\preceq, \varphi)} \qquad \psi \rightarrow \theta \coloneqq \overline{\mathsf{fp}[\preceq, \mathsf{PG}(\{\psi, \overline{\theta}\}))}$$

2. Modal fragment: $\mathbb{JM}$-elimination rule for $\langle , \rangle$ and $[\,]$:

   (a) For modality $\langle \mathrm{r} \rangle$ $$\langle \mathrm{r} \rangle \varphi \coloneqq \mathsf{fp}[\mathrm{r}, \varphi)$$

   (b) For modality $[\mathrm{r}]$ $$[\mathrm{r}] \varphi \coloneqq \overline{\mathsf{fp}[\preceq, (\mathsf{fp}[\mathrm{r}, \overline{\varphi})))}$$

This will give a graphical converter for $\mathbb{JM}$ (see p. 33)

5 General Converter and Model-Checker

General graphical converter $\mathtt{GC}$: (see p. 27)

generates a converter $\mathtt{GC}_{\mathrm{Lg}}$, for each logic Lg

$\mathtt{GC}_{\mathrm{Lg}}$ eliminates logical symbols producing neat expressions

Neat expressions $\quad \mathrm{F} ::= \mathrm{p} \mid \overline{\mathrm{F}} \mid \mathsf{PG}(\boldsymbol{F}) \mid \mathsf{BK}[\boldsymbol{F}] \mid \mathsf{FP}[\mu, \boldsymbol{F}) \qquad \mathrm{p} \in \mathsf{PL}$

$$\boldsymbol{F} ::= \emptyset \mid \boldsymbol{F} \cup \{\mathrm{F}\} \qquad (\boldsymbol{F} = \{\mathrm{F}_1, \ldots, \mathrm{F}_h\})$$

General converters

Formula φ

↓

$\left(\text{Elimination rules of Lg} : \text{ER}_{\text{Lg}} \right) \longrightarrow$ General converter GC

↓

Expression E

Converter for Lg

$$\mathtt{GC} : \text{Logic} \to (\text{Formulas} \to \text{Neat Expressions})$$

$$(\mathrm{p})\quad \mathtt{GC}_{\mathrm{Lg}}(\mathrm{p}) \coloneqq \mathrm{p} \qquad \text{(convert propositional letter } \mathrm{p} \in \mathrm{PL}\text{)}$$

$$(\dagger)\quad \mathtt{GC}_{\mathrm{Lg}}(\dagger) \coloneqq \vec{\dagger}_{\mathrm{Lg}} \qquad \text{(eliminate 0-ary } \dagger\text{)}$$

$$(\nabla)\quad \mathtt{GC}_{\mathrm{Lg}}(\nabla\varphi) \coloneqq \vec{\nabla}_{\mathrm{Lg}}[\mathtt{GC}_{\mathrm{Lg}}(\varphi)] \qquad \text{(eliminate 1-ary } \nabla\text{)}$$

$$(\bullet)\quad \mathtt{GC}_{\mathrm{Lg}}(\psi \bullet \theta) \coloneqq \vec{\bullet}_{\mathrm{Lg}}[\mathtt{GC}_{\mathrm{Lg}}(\psi), \mathtt{GC}_{\mathrm{Lg}}(\theta)] \qquad \text{(eliminate 2-ary } \bullet\text{)}$$

$$(\mu)\quad \mathtt{GC}_{\mathrm{Lg}}(\mu\varphi) \coloneqq \vec{\mu}_{\mathrm{Lg}}[\mathtt{GC}_{\mathrm{Lg}}(\varphi)] \qquad \text{(eliminate modality } \mu\text{)}$$

function $\mathbb{GC}$ (Lg : Logic , φ : Formula) $\rightarrow$ Expression

case$[\varphi]$

$\varphi \in \mathrm{PL}$	**return**	φ
$\varphi = \dagger$	**return**	$\vec{\dagger}_{\mathrm{Lg}}$
$\varphi = \nabla\theta$	**return**	$\vec{\nabla}_{\mathrm{Lg}}[\mathbb{GC}(\theta)]$
$\varphi = \psi \bullet \theta$	**return**	$\vec{\bullet}_{\mathrm{Lg}}[\mathbb{GC}(\psi), \mathbb{GC}(\theta)]$
$\varphi = \mu\psi$	**return**	$\vec{\mu}_{\mathrm{Lg}}[\mathbb{GC}(\psi)]$

esac(φ)

end-function $\mathbb{GC}$. (cf. p. 28)

Specific converters instantiated graphical converter

Formula φ $\longrightarrow$ [ER_{Lg} $\rightarrow$ GC] $\longrightarrow$ Neat Expression E

Instantiated Graphical Converter $\mathtt{GC}_{\mathtt{KM}}$ for Classical modal logic $\mathtt{KM}$

$\mathtt{GC}_{\mathtt{KM}}$: ML-formula $\rightarrow$ Neat expression

$(p)\quad \mathtt{GC}_{\mathtt{KM}}(p) \;:=\; p$ — propositional letter

$(\bot)\quad \mathtt{GC}_{\mathtt{KM}}(\bot) \;=\; \vec{\bot}_{\mathtt{KM}} \;=\; \{\ \}$ — empty book $\mathsf{BK}[\emptyset]$

$(\neg)\quad \mathtt{GC}_{\mathtt{KM}}(\neg\varphi) \;=\; \vec{\neg}_{\mathtt{KM}}[\mathtt{GC}_{\mathtt{KM}}(\varphi)] \;=\; \overline{\mathtt{GC}_{\mathtt{KM}}(\varphi)}$

$(\wedge)\quad \mathtt{GC}_{\mathtt{KM}}(\psi\wedge\theta) \;=\; \vec{\wedge}_{\mathtt{KM}}[\mathtt{GC}_{\mathtt{KM}}(\psi),\mathtt{GC}_{\mathtt{KM}}(\theta)] \;=\; \mathsf{PG}(\{\mathtt{GC}_{\mathtt{KM}}(\psi),\mathtt{GC}_{\mathtt{KM}}(\theta)\})$

$(\vee)\quad \mathtt{GC}_{\mathtt{KM}}(\psi\vee\theta) \;=\; \vec{\vee}_{\mathtt{KM}}[\mathtt{GC}_{\mathtt{KM}}(\psi),\mathtt{GC}_{\mathtt{KM}}(\theta)] \;=\; \mathsf{BK}[\{\mathtt{GC}_{\mathtt{KM}}(\psi),\mathtt{GC}_{\mathtt{KM}}(\theta)\}]$

$(\rightarrow)\quad \mathtt{GC}_{\mathtt{KM}}(\psi\rightarrow\theta) \;=\; \vec{\rightarrow}_{\mathtt{KM}}[\mathtt{GC}_{\mathtt{KM}}(\psi),\mathtt{GC}_{\mathtt{KM}}(\theta)] \;=\; \mathsf{BK}[\{\overline{\mathtt{GC}_{\mathtt{KM}}(\psi)},\mathtt{GC}_{\mathtt{KM}}(\theta)\}]$

$(\langle r\rangle)\quad \mathtt{GC}_{\mathtt{KM}}(\langle r\rangle\varphi) \;=\; \vec{\langle r\rangle}_{\mathtt{KM}}[\mathtt{GC}_{\mathtt{KM}}(\varphi)] \;=\; \mathsf{fp}[r,\mathtt{GC}_{\mathtt{KM}}(\varphi))$

$([r])\quad \mathtt{GC}_{\mathtt{KM}}([r]\varphi) \;=\; \vec{[r]}_{\mathtt{KM}}[\mathtt{GC}_{\mathtt{KM}}(\varphi)] \;=\; \overline{\mathsf{fp}[r,\overline{\mathtt{GC}_{\mathtt{KM}}(\varphi)})}$

Exmpl. $\mathtt{GC}_{\mathtt{KM}}(\langle r\rangle[s]\neg p) \;=\; \mathsf{fp}[r,(\mathtt{GC}_{\mathtt{KM}}([s]\neg p))) \;=\; \mathsf{fp}[r,(\overline{\mathsf{fp}[s,\overline{\overline{p}})}))$ ♮

Instantiated Graphical Converter $\mathtt{GC}_{\mathtt{KM}}$ for Classical modal logic $\mathtt{KM}$

function $\mathtt{GC}_{\mathtt{KM}}$ (φ : ML-formula) $\to$ Neat expression

case[φ]

$\varphi \in \mathsf{PL}$	**return**	φ
$\varphi = \bot$	**return**	$\{\ \}$
$\varphi = \neg\theta$	**return**	$\overline{\mathtt{GC}_{\mathtt{KM}}(\theta)}$
$\varphi = \psi \wedge \theta$	**return**	$\mathsf{PG}(\{\mathtt{GC}_{\mathtt{KM}}(\psi), \mathtt{GC}_{\mathtt{KM}}(\theta)\})$
$\varphi = \psi \vee \theta$	**return**	$\mathsf{BK}[\{\mathtt{GC}_{\mathtt{KM}}(\psi), \mathtt{GC}_{\mathtt{KM}}(\theta)\}]$
$\varphi = \psi \to \theta$	**return**	$\mathsf{BK}[\{\overline{\mathtt{GC}_{\mathtt{KM}}(\psi)}, \mathtt{GC}_{\mathtt{KM}}(\theta)\}]$
$\varphi = \langle r \rangle \psi$	**return**	$\mathsf{fp}[r, \mathtt{GC}_{\mathtt{KM}}(\psi))$
$\varphi = [r]\psi$	**return**	$\overline{\mathsf{fp}[r, \overline{\mathtt{GC}_{\mathtt{KM}}(\psi)})}$

esac(φ)

end-function $\mathtt{GC}_{\mathtt{KM}}$. (cf. p. 31)

Exmpl. $\mathtt{GC}_{\mathtt{KM}}(\langle r \rangle [s] \neg p) = \mathsf{fp}[r, (\mathtt{GC}_{\mathtt{KM}}([s]\neg p))) = \mathsf{fp}[r, (\overline{\mathsf{fp}[s, \overline{\overline{p}})}))$ ♮

Instantiated Graphical Converter $\mathtt{GC}_{\mathbb{JM}}$ for Intuitionistic modal logic $\mathbb{JM}$

$\mathtt{GC}_{\mathbb{JM}}$: ML-formula $\to$ Neat expression

($\star$) For $p, \bot, \wedge, \vee$ and $\langle r \rangle$ as for classical modal logic

$(\neg)$ $\mathtt{GC}_{\mathbb{IP}}(\neg\varphi) = \vec{\neg}_{\mathbb{JM}}[\mathtt{GC}_{\mathbb{JM}}(\varphi)] = \overline{\mathrm{fp}[\preceq, \mathtt{GC}_{\mathbb{JM}}(\varphi))}$

$(\to)$ $\underline{\mathtt{GC}_{\mathbb{IP}}(\psi{\to}\theta) = \vec{\to}_{\mathbb{JM}}[\mathtt{GC}_{\mathbb{JM}}(\psi), \mathtt{GC}_{\mathbb{JM}}(\theta)] =}$
$\mathrm{fp}[\preceq, \mathrm{PG}(\{\mathtt{GC}_{\mathbb{JM}}(\psi), \overline{\mathtt{GC}_{\mathbb{JM}}(\theta)}\}))$

$([r])$ $\mathtt{GC}_{\mathbb{JM}}([r]\varphi) = \vec{[r]}_{\mathbb{JM}}[\mathtt{GC}_{\mathbb{JM}}(\varphi)] = \overline{\mathrm{fp}[\preceq, (\mathrm{fp}[r, \overline{\mathtt{GC}_{\mathbb{JM}}(\varphi)}))))}$

Exmpl. $\mathtt{GC}_{\mathbb{JM}}(\langle r \rangle [s] \neg p) = \mathrm{fp}[r, \overline{(\mathrm{fp}[\preceq, (\mathrm{fp}[s, \overline{\overline{\mathrm{fp}[\preceq, p)}}))))}).$ $\natural$

Instantiated Graphical Converter $\mathbb{GC}_{\mathbb{JM}}$ for Intuitionistic modal logic $\mathbb{JM}$

function $\mathbb{GC}_{\mathbb{JM}}$ (φ : ML-formula) $\rightarrow$ Neat expression

case$[\varphi]$

$\varphi \in \mathbb{PL}$	**return**	φ
$\varphi = \bot$	**return**	$\{\ \}$
$\varphi = \neg\theta$	**return**	$\overline{\mathrm{fp}[\preceq, \mathbb{GC}_{\mathbb{JM}}(\varphi))}$
$\varphi = \psi \wedge \theta$	**return**	$\mathbb{PG}(\{\mathbb{GC}_{\mathbb{JM}}(\psi), \mathbb{GC}_{\mathbb{JM}}(\theta)\})$
$\varphi = \psi \vee \theta$	**return**	$\mathsf{BK}[\{\mathbb{GC}_{\mathbb{JM}}(\psi), \mathbb{GC}_{\mathbb{JM}}(\theta)\}]$
$\varphi = \psi \rightarrow \theta$	**return**	$\overline{\mathrm{fp}[\preceq, \mathbb{PG}(\{\mathbb{GC}_{\mathbb{JM}}(\psi), \overline{\mathbb{GC}_{\mathbb{JM}}(\theta)}\}))}$
$\varphi = \langle r \rangle \psi$	**return**	$\mathrm{fp}[r, \mathbb{GC}_{\mathbb{JM}}(\psi))$
$\varphi = [r]\,\psi$	**return**	$\overline{\mathrm{fp}[\preceq, (\mathrm{fp}[r, \overline{\mathbb{GC}_{\mathbb{JM}}(\psi)}))}$

esac(φ)

end-function $\mathbb{GC}_{\mathbb{JM}}$. (cf. p. 33)

General graphical model-checker

$\mathtt{GMc}$: (Model $\mathfrak{M}$, State a, Neat expression E] $\rightarrow$ **Bln**

function $\mathtt{GMc}$ (a : State , E : Neat expression) $\rightarrow$ **Bln** (for short)

case[E]

$E \in PL$	**if** $a \in E^{\mathfrak{M}}$ **then** **return** **true** **else** **return** **false** **fi**
$E = \overline{F}$	**return** **not** $\mathtt{GMc}(a : F]$
$E = PG(\boldsymbol{F})$	**return** **and**$(\{\mathtt{GMc}(a : F] \,/\, F \in \boldsymbol{F}\})$
$E = BK[\boldsymbol{F}]$	**return** **or**$[\{\mathtt{GMc}(a : F] \,/\, F \in \boldsymbol{F}\}]$
$E = FP[\mu, \boldsymbol{F})$	**return** **or**$[\{\mathtt{GMc}(b : PG(\boldsymbol{F})] \,/\, b \in [a^{\mu})_{\mathfrak{M}}\}]$
esac(E)	$[a^{\mu})_{\mathfrak{M}} \coloneqq \{b \in M \,/\, (a, b) \in \mu^{\mathfrak{M}}\}$
	states reached under a transition

Graphical model-checker for logic Lg

$$\mathtt{GMc}_{Lg}(\mathfrak{M}, a : \varphi] \coloneqq \mathtt{GMc}(\mathfrak{M}, a : GC_{Lg}(\varphi))$$

Exmpl.

Consider model $\mathfrak{C}$

$$q \supset\!\!\rightsquigarrow \mathsf{b} \overset{r}{\twoheadleftarrow\!\!\rightsquigarrow} \mathsf{a} \overset{r}{\rightsquigarrow\!\!\twoheadrightarrow} \mathsf{c} \overset{s}{\rightsquigarrow\!\!\twoheadrightarrow} \mathsf{d} \rightsquigarrow\!\!\subset p$$

Check whether $\varphi = \langle r \rangle [s] \neg p$ holds at state a of model $\mathfrak{C}$

¡

$$\mathtt{GC}_{\mathtt{KM}}(\varphi) = \mathrm{fp}[r, (\overline{\mathrm{fp}[s, \overline{\overline{p}})}))$$

Model-checker $\mathtt{GMc}(a : \varphi]$ operation (GMc(a : φ] short for $\mathtt{GMc}_{KM}(\mathfrak{C}, a : \varphi])$)

$$\mathtt{GMc}(a : \varphi] \quad = \quad \mathtt{GMc}(a : \mathrm{fp}[r, (\overline{\mathrm{fp}[s, \overline{\overline{p}}])}])]$$

$[a^r)_{\mathfrak{M}} = \{b, c\}$ $\quad || \; (\overset{r}{\mapsto})$

$$\mathtt{GMc}(b : \overline{\mathrm{fp}[s, \overline{\overline{p}})}] \quad \textbf{or} \quad \mathtt{GMc}(c : \overline{\mathrm{fp}[s, \overline{\overline{p}})}]$$

$|| \; (\overline{\ })$ $\qquad || \; (\overline{\ })$

$$\underline{\textbf{not}}\ \mathtt{GMc}(b : \mathrm{fp}[s, \overline{\overline{p}})] \qquad \underline{\textbf{not}}\ \mathtt{GMc}(c : \mathrm{fp}[s, \overline{\overline{p}})]$$

$[c^s)_{\mathfrak{M}} = \{d\}$ $\qquad || \; (\overset{s}{\mapsto})$

$[b^s)_{\mathfrak{M}} = \emptyset$ $\qquad || \; (\overset{s}{\mapsto})$ $\qquad \underline{\textbf{not}}\ \mathtt{GMc}(d : \overline{\overline{p}}]$

$|| \; (\overline{\ })^2$

$$\underline{\textbf{not}}\ \textbf{false} \qquad \vdots \qquad \underline{\textbf{not}}\ \mathtt{GMc}(d : p]$$

$p^{\mathfrak{C}} = \{d\}$ $\qquad || \; (p)$

$||$ $\qquad \underline{\textbf{not}}\ \textbf{true}$

$||$

$$\textbf{true} \quad \underline{\textbf{or}} \quad \textbf{false}$$

$||$

true

6 Concluding remarks

- General graphical approach to specification and construction of model-checkers (cf. p. 5)
- General model-checkers that can be instantiated (cf. p. 6)

2 components (cf. p. 6):

1. converter — logic-dependent — user's concern (cf. p. 28)
2. general model checker — automatic operation (cf. p. 35)

Examples

Classical modal logic

Intuitionistic modal logic

Further relational constants and special modalities :

Reachable set $[a^{\mu})_{\mathfrak{M}}$ for some modalities

	Modality $\langle t \rangle$	Special modalities
Relation name $r \in \mathtt{RN}$	$\{ b \in M \,/\, (a, b) \in r^{\mathfrak{M}} \}$	Universal E
Constant null 0	$\emptyset$	$[a^{E})_{\mathfrak{M}} = M$
Constant square $\bowtie$	M	
Constant identity ι	$\{ a \}$	Difference D
Constant diversity ∂	$M \setminus \{ a \}$	$[a^{D})_{\mathfrak{M}} = M \setminus \{ a \}$

Extensions

Exmpl.

– Structured modalities

composition & intersection

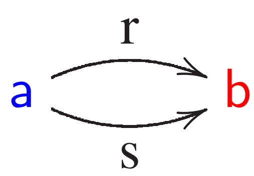
- $r ; s$

$$a \xrightarrow{r ; s} b \text{ iff } a \xrightarrow{r} c \xrightarrow{s} b$$

new node c

- $r \sqcap s$

$$a \xrightarrow{r \sqcap s} b \text{ iff }$$



– tests $\langle\psi?\rangle$ and $[\psi?]$ in PDL

eliminate	by	
$\langle\psi?\rangle\,\theta$	$\psi \supset\!\!- - - \widehat{x} - - -\!\!\subset \theta$	$= \mathrm{PG}(\{\psi,\theta\})$
$[\psi?]\,\theta$	$\left\{ \begin{array}{c} \widehat{x} - - -\!\!\subset \overline{\psi} \\ \widehat{x} - - -\!\!\subset \theta \end{array} \right\}$	$= \mathrm{BK}[\{\overline{\psi},\theta\}]$

Model $\mathfrak{M}$

Universe $\mathsf{M} \neq \emptyset$

$$\begin{cases} \text{subset } \varphi^{\mathfrak{M}} \subseteq \mathsf{M} & \varphi \in \Phi \\ \text{2-ary relation } \mu^{\mathfrak{M}} \text{ on } \mathsf{M} & \mu \in \Xi \end{cases}$$

Exmpl. Classical Modal Logic Model $\mathfrak{C}$ (subset $\mathrm{p}^{\mathfrak{C}} \subseteq \mathsf{M}$, $\mathrm{p} \in \mathrm{PL}$)

$$\mathsf{M} = \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{d}\}$$

$$\mathrm{p}^{\mathfrak{C}} = \{\mathsf{d}\} \qquad \mathrm{q}^{\mathfrak{C}} = \{\mathsf{b}\}$$

$$\mathrm{s}^{\mathfrak{C}} = \{(\mathsf{c}, \mathsf{d})\} \qquad \mathrm{r}^{\mathfrak{C}} = \left\{ \begin{array}{c} (\mathsf{a}, \mathsf{b}) \\ (\mathsf{a}, \mathsf{c}) \end{array} \right\}$$

$$\mathrm{q} \supset \sim\sim\sim\; \mathsf{b} \overset{\mathrm{r}}{\leftsquigarrow} \mathsf{a} \overset{\mathrm{r}}{\rightsquigarrow} \mathsf{c} \overset{\mathrm{s}}{\rightsquigarrow} \mathsf{d} \;\sim\sim\sim \subset \mathrm{p}$$

Intuitionistic modal model $\mathfrak{J}$