

A Quick Introduction to Research Topics at Tsinghua

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Outline

- 1 The Logic Center
- 2 My own research
 - Preference Logic
 - Social Epistemic Logic
 - Graph Game Logics
 - Logic Interfacing the current AI
- 3 Research of Junhua Yu
- 4 Research of Chenwei Shi

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Reason-based preference

Definition 3 (lexicographic preference)

Given a priority sequence of length n , two objects x and y ,
Pref(\mathbf{x}, \mathbf{y}) is defined as follows:

$$Pref_1(x, y) ::= C_1(x) \wedge \sim C_1(y),$$

$$Pref_{k+1}(x, y) ::=$$

$$Pref_k(x, y) \vee (Eq_k(x, y) \wedge C_{k+1}(x) \wedge \sim C_{k+1}(y)), k < n,$$

$Pref(x, y) ::= Pref_n(x, y)$, with the *auxiliary binary predicate*

$$Eq_k(x, y) = (C_1(x) \leftrightarrow C_1(y)) \wedge \cdots \wedge (C_k(x) \leftrightarrow C_k(y)).$$

Reason-based preference change

Preference change via *priority change*.

Four operations considered, and their logical laws determined:

$[^+A]$ of adding A to the right

$[A^+]$ of adding A to the left

$[-]$ of dropping the last element of a priority sequence

$[i \leftrightarrow i+1]$ of interchanging the i -th and $i+1$ -th elements.

Recursion axioms

$$[{}^+A]Pref(x, y) \leftrightarrow Pref(x, y) \vee (Eq(x, y) \wedge A(x) \wedge \sim A(y))$$

$$[A^+]Pref(x, y) \leftrightarrow ((A(x) \wedge \sim A(y)) \vee ((A(x) \leftrightarrow A(y)) \wedge Pref(x, y)))$$

$$[-]Pref(x, y) \leftrightarrow Pref_{n-1}(x, y)$$

$$[i \leftrightarrow i + 1]Pref(x, y) \leftrightarrow Pref_{i-1}(x, y) \vee (Eq_{i-1}(x, y) \wedge A_{i+1}(x) \wedge \sim A_{i+1}(y)) \vee (Pref_i(x, y) \wedge (A_{i+1}(x) \leftrightarrow A_{i+1}(y))) \vee (Eq_{i+1}(x, y) \wedge Pref(x, y)).$$

Relating two dynamics

Definition 4

Let $\alpha: (\mathcal{G}, A) \rightarrow \mathcal{G}'$, with $\mathcal{G}, \mathcal{G}'$ priority graphs, and A a new proposition not in \mathcal{G} . Let $\sigma: (\preceq, A) \rightarrow \preceq'$ be a map with \preceq, \preceq' preference relations over worlds. We say that α **induces** σ , if:

$$\sigma(\preceq_{\mathcal{G}}, A) = \preceq_{\alpha(\mathcal{G}, A)}$$

A graph operation α is *PDL-definable* if it induces a preference transformer σ that is *PDL-definable* in the earlier format.

Harmony between two levels

Fact 5

Prefixing a new proposition A to a priority graph $(\mathcal{G}, <)$ induces the 'promotion' operation $\uparrow A$ on possible worlds models.

More precisely, the following diagram commutes:

$$\begin{array}{ccc} \langle \mathcal{G}, < \rangle & \xrightarrow{A; \mathcal{G}} & \langle (A; \mathcal{G}), < \rangle \\ \downarrow & & \downarrow \\ \langle W, \preceq \rangle & \xrightarrow{\uparrow A} & \langle W, \uparrow A(\preceq) \rangle \end{array}$$

This shows that some changes at the richer priority level can be tracked faithfully at the poor preference level.

Obstacles to a complete match

Fact 6

The deletion operation $del(\mathcal{G})$ is not PDL-trackable.

Fact 7

Not all PDL-definable operations are graph-definable.

In general then, both levels, preference order of worlds and priority order of criteria, have their own intrinsic dynamics of preference change, supporting separate logical systems.

Further development and related works

- When facing uncertainty about whether an object possesses certain properties, we need to introduce belief to define preference; see my attempt in the book.
- Models in rational choice theory that extend our approach to first-order logic, probability, and numerical utility, e.g.
 - Dietrich, F. and List, C., 2013, 'Where do preferences come from?'. *International Journal of Game Theory*, 42(3): 613-637.
 - Osherson, D. and Weinstein, S., 2012, 'Preference based on reasons', *The Review of Symbolic Logic*, 5(1):122-147.

Further development and related works

Logics for game theory with priority structure, preference change in social setting, e.g.

- Grossi, D. and P. Turrini, 2012, ‘Short-sight in extensive games’. In V. Conitzer and M. Winikoff, eds, *Proceedings of the 11th AAMAS*.
- Liu, C., Liu, F., Su, K., and Zhu, E., 2016, ‘A logical characterization of extensive games with short sight’, *Theoretical Computer Science*, Vol. 612, 2016, Pages 63-82.
- Liang, Z. and Seligman, J., 2011, ‘The dynamics of peer pressure’, in *Proceedings of LORI 2011*, Berlin, Heidelberg: Springer, vol. 6953 of LNCS, 377–388

Further development and related works

Priorities structures in formal ethics and deontic logic, e.g.

- van Benthem, J., D. Grossi, and F. Liu, 2014, ‘Priority structures in deontic logic’. *Theoria*, 80(2), pages 116–152.
- Osherson and Weinstein, 2014, Deontic modality based on preference.
- Xie, K., Yan, J. A logic for desire based on causal inference, *Journal of Logic and Computation*, Vol. 34, Issue 2, 2024.

F. Liu and L. van der Torre, Preference Logic, *Stanford Encyclopaedia in Philosophy*, 2025.

II. Social Epistemic Logic

- Reasoning about knowledge and **social relations** (high rationality approach)
- Modelling **social influence** and group belief (low rationality approach)
- Both are needed to get the real picture of social agency, but they also raise a fundamental conceptual and practical issue of interfacing.

Epistemic logic of friendship: syntax and semantics

Formulas

$\varphi ::= p \mid \eta \mid \sim\varphi \mid (\varphi \& \varphi) \mid K\varphi \mid F\varphi \mid A\varphi \mid \downarrow\eta\varphi$
($p \in \text{Prop}$, $\eta \in \text{ANom}$)

Models

$M = \langle W, A, k, f, g, V \rangle$

k_a is an equivalence relation on W for each $a \in A$

f_w is an irreflexive, symmetric relation on A for each $w \in W$

$g(\eta) \in A$ for each $\eta \in \text{ANom}$

$V(p) \subseteq W \times A$ for each $p \in \text{Prop}$

Epistemic logic of friendship: syntax and semantics

$M, w, a \models \eta$	iff	$g(\eta) = a$, for $\eta \in \text{ANom}$
$M, w, a \models K\varphi$	iff	$M, v, a \models \varphi$ for every $v \in W$ such that $k_a(w, v)$
$M, w, a \models F\varphi$	iff	$M, w, b \models \varphi$ for every $b \in A$ such that $f_w(a, b)$
$M, w, a \models A\varphi$	iff	$M, w, b \models \varphi$ for every $b \in A$.
$M, w, a \models \Delta\varphi$	iff	$\Delta M, \Delta w, a \models \varphi$

where Δ is one of a range of dynamic operators

Some results

Theorem 8 (Sano 2017)

There is a Hilbert style sound and complete axiomitization for EFL without \downarrow w.r.t the two-dimensional semantics.

[translate a sound, complete and cut-free tree sequent calculus for EFL into an ordinary formula to specify a Hilbert system.]

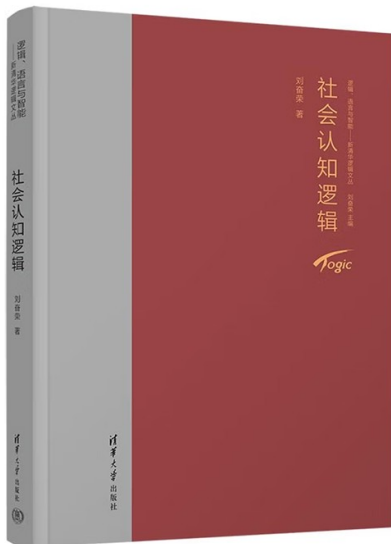
Theorem 9 (Liang 2020)

There is a Hilbert style sound and complete axiomitization for the full EFL with \downarrow .

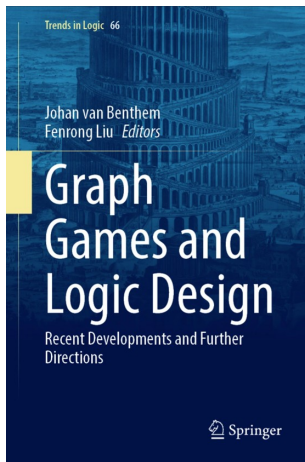
Selected works

- Seligman, Liu, and Girard, Facebook and Epistemic Logic of Friendship, in TARK2013 Proceedings. pp. 229-238.
- Liu, Seligman, and Girard, Logical Dynamics of Belief Change in the Community, *Synthese*, 191(11): 2403-2431, 2014.
- Christoff, Dynamic Logics of Networks, Ph.d dissertation, ILLC, University of Amsterdam 2016.
- Xue, In Search of Homo Sociologicus, Ph.d dissertation, The Graduate Center, City University of New York, 2017.
- Shi, *Reason to Believe*, ILLC, University of Amsterdam, Ph.D Dissertation, 2018.

Social Epistemic Logic, 2023



III. Graph Game Logics



Modal Logic of Cop-and-Robber

We proposed a two-dimensional modal logic **LCR** that can study the interaction between two players on the graph.

Definition 10 (Language)

Let P_R and P_C denote two countable sets of propositional variables. The language is given as follows:

$$\varphi ::= p_R \mid p_C \mid I \mid \sim\varphi \mid (\varphi \wedge \varphi) \mid \langle R \rangle\varphi \mid \langle C \rangle\varphi$$

where $p_R \in P_R$, $p_C \in P_C$, and I is a propositional constant.

\top , \perp , \rightarrow , \vee are defined as usual.

Also, $R\varphi := \sim\langle R \rangle\sim\varphi$ and $C\varphi := \sim\langle C \rangle\sim\varphi$

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