# THE RING OF ISOBARIC POLYNOMIALS AND HYPERBOLIC GEOMETRY

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An isobaric polynomial is a symmetric polynomial written on the Elementary Symmetric Polynomial basis [1]. These polynomials have the pleasant property of being indexed by partitions of the natural numbers, and they turn out to provide useful representations of several arithmetic and algebraic structures, [4], [5].. We introduce two new operators, the Isobaric logarithm and the isobaric exponential operator from which we can define the hyperbolic trigonometric functions: the isobaric hyperbolic sine, cosine, tangent, etc. for which all of the usual hyperbolic trigonometric identities are satisfied. Since the ring of isobaric polynomials is isomorphic to the ring of symmetric polynomials, all of these identities induce (new) identities in the ring of symmetric polynomials, and imply the existence of a hyperbolic geometric structure for the ring of symmetric polynomials. We are dealing with an algebra (or rather two isomorphic algebras), which has the property of supporting a hyperbolic geometry. This suggests the question: which linear algebras are capable of supporting a geometric structure, a question similar to the one answered by the Geometrization Theorem due to William Thurston, in which it is proved that every 3-dimensional manifold suitably restricted supports at least one of eight kinds of geometries. The results of this paper for our question show that for a positive answer, it is sufficient that such algebra have an invertible logarithmic operator. In this paper we show how a hyperbolic trigonometry is constructed in the ring of isobaric polynomials. This ring has two especially interesting bases, directly related to arithmetic number theory; namely, the set of Generalized Fibonacci Polynomials (GFP) and the set of Generalized Lucas Polynomials (GLP).

We define a suitable convolution product of two isobaric polynomials of the same isobaric degree .This convolution product gives the isobaric ring a product structure and can be mapped faithfully to the Dirichlet product in the ring (UFD) of multiplicative arithmetic functions. This map can be extended to a map which produces a faithful representation of the subgroup of Multiplicative Arithmetic Functions, and, using the GLPs instead of the GFPs, to a faithful representation of the subgroup of additive arithmetic functions. We produce the hyperbolic trigonometry in the following way: We first introduce the Isobaric Logarithm L, and show that it does, in fact, satisfy the defining relations for a logarithm and that it has an inverse, which is our Exponential Operator. It turns out that the logarithms of the GFPs are the GLPs, and that, as expected, the images of the Exponential operator are the GFPs. We use the exponential operator in the standard way to define the isobaric trigonometric functions: Isocosh, Isosinh, Isotanh,etc, which we prove satisfy all of the standard identities for the hyperbolic trigonometric functions: sum formulae, difference of squares of *cosh* and *sinh* equal to 1, etc. The remaining hyperbolic functions are defined in the usual way. This construction then implies the existence of a hyperbolic geometry of isobaric polynomials, Moreover, the isomorphism of the isobaric ring with the ring of symmetric polynomials yields a wealth of new identities for symmetric polynomials, and implies additional structure for the group of multiplicative arithmetic functions and the group of additive arithmetic functions.

References:

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